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Political Selection and Persistence of Bad Governments*

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Abstract

We study dynamic selection of governments under different political institutions, with a special focus on institutional "flexibility". A government consists of a subset of the individuals in the society. The competence level of the government in office determines collective utilities (e.g., by determining the amount and quality of public goods), and each individual derives additional utility from being part of the government (e.g., corruption or rents from holding office). We characterize dynamic evolution of governments and determine the structure of stable governments, which arise and persist in equilibrium. Perfect democracy, where current members of the government do not have an incumbency advantage or special powers, always leads to the emergence of the most competent government. However, any deviation from perfect democracy destroys this result. There is always at least one other, less competent government that is also stable and can persist forever, and even the least competent government can persist forever in office. Moreover, a greater degree of democracy may lead to worse governments. In contrast, in the presence of stochastic shocks or changes in the environment, greater democracy corresponds to greater flexibility and increases the probability that high competence governments will come to power. This result suggests that a particular advantage of democratic regimes may be their greater adaptability to changes rather than their performance under given conditions. Finally, we show that, in the presence of stochastic shocks, "royalty-like" dictatorships may be more successful than "junta-like" dictatorships, because they might also be more adaptable to change.

Keywords: institutional flexibility, quality of governance, political economy, political transitions, voting.

JEL Classification: D71, D74, C71.

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1 Introduction

A central role of (successful) political institutions is to ensure the selection of the right (honest, competent, motivated) politicians. Besley (2005, p. 43), for example, quotes James Madison, to emphasize the importance of the selection of politicians for the success of a society:

> "The aim of every political Constitution, is or ought to be, first to obtain for rulers men who possess most wisdom to discern, and most virtue to pursue, the common good of society; and in the next place, to take the most effectual precautions for keeping them virtuous whilst they continue to hold their public trust."

Equally important, but less often stressed, is the "flexibility" of institutions, meaning their ability to deal with shocks and changing situations, for example by adapting the nature of the government and changing the characteristics of those in power in response to changes in the environment.[1] In this paper, we construct a simple dynamic model of government formation to highlight the potential sources of inefficiency in the selection of governments and to identify features of political processes that create "institutional flexibility".[2]

The "government" is made up of a subset of the citizens (e.g., each three-player group may be a government, etc.). Each (potential) government has a different level of competence, determining the collective utility it provides to citizens (e.g., the level of public goods). Each individual also receives rents from being part of the government (additional income, utility of office, or rents from corruption). New governments are put in place by a combination of "votes" from the citizens and "consent" from current government members. The extent of necessary consent of current government members is a measure of the "degree of democracy". For example, a perfect democracy can be thought of as a situation in which current incumbents have no special power and no such consent is necessary. Many political institutions, in contrast, provide additional decision-making or blocking power to current government members. For instance, in many democracies, various sources of incumbency advantage make the government in power harder to oust than instituting it anew would have been had it been out of power (e.g., Cox and Katz, 1996, for a discussion of such incumbency advantage in mature democracies).

[1] For instance, the skills necessary for successful wartime politicians and governments are very different from those that are useful for the successful management of the economy during peacetime, as illustrated perhaps most clearly by Winston Churchill's political career.

[2] Even though we model changes in the underlying environment and the competences of different governments as resulting from stochastic shocks, in practice these may also result from deterministic changes in the nature of the economy. For example, authoritarian regimes such as the rule of General Park in South Korea or Lee Kuan Yew in Singapore may be beneficial or less damaging during the early stages of development, while a different style of government, with greater participation, may be necessary as the economy develops and becomes more complex. Acemoglu, Aghion and Zilibotti (2006) suggest that "appropriate" institutions may be a function of the distance of an economy to the world technology frontier, and Aghion, Alesina and Trebbi (2009) provide empirical evidence consistent with this pattern.

In nondemocratic societies, this advantage of current government members is more pronounced, even palpable: only new governments that include some members of previous governments might be feasible (as, unfortunately, illustrated by the recent events in Zimbabwe or Iran). While the degree of incumbency advantage is not the only characteristic of democracy, focusing on this specific metric allows a simple parameterization of different regimes, ranging from personalistic dictatorship to representative democracy, and thus leads to informative comparisons between these regimes both in stochastic and nonstochastic environments.

The first contribution of our paper is to provide a general and tractable framework for the study of such dynamic political selection issues and to provide a detailed characterization of the structure (and efficiency) of the selection of politicians under different political institutions. Perfect democracies always ensure the emergence of the best (most competent) government. In contrast, under any other arrangement, incompetent and bad governments can emerge and persist despite the absence of information-related challenges to selecting good politicians. For example, even a small departure from perfect democracy, whereby only one member of the current government needs to consent to a new government, may make the *worst* possible government persist forever. The intuitive explanation for why even a small degree of incumbency advantage might lead to such outcomes is as follows: improvements away from a bad (or even the worst) government might lead to another potential government that is itself unstable and will open the way for a further round of changes. If this process ultimately leads to a government that does not have any common members with the initial government, then it may fail to get the support of any of the initial government members. In this case, the initial government survives even though it has a low, or even possibly the lowest, level of competence. This result provides a potential explanation for why many autocratic or semi-autocratic regimes, including those currently in power in Iran, Russia, Venezuela and Zimbabwe, resist the inclusion of "competent technocrats" in the government—because they are afraid that these technocrats can later become supporters of further reform, ultimately unseating even the most powerful current incumbents.[3]

Another important implication of these dynamic interactions in political selection is that, beyond perfect democracy, there is no obvious ranking among different shades of imperfect democracy (and dictatorships). Any of these different regimes may lead to better governments in the long run. This result is consistent with the empirical findings in the literature that show no clear-cut relationship between democracy and economic performance (e.g., Przeworski and Limongi, 1997, Barro, 1997, Minier, 1999). In fact, both under imperfect democracies and extreme dictatorships, the competence of the equilibrium government and the success of

[3] For example, on Iranian politics and resistance to the inclusion of technocrats during Khomeini's reign, see Menashri (2001), and more recently under Ahmadinejad's presidency, see Alfoneh (2008). On Russian politics under Vladimir Putin, see Baker and Glasser (2007). On Zimbabwe under Mugabe, see Meredith (2007).

the society depend strongly on the identity of the initial members of the government. This is consistent with the emphasis in the recent political science and economics literatures on the role that leaders may play under weak institutions (see, for example, Brooker, 2000, or Jones and Olken, 2004, who show that the death of an autocrat leads to a significant change in growth, and this does not happen with democratic leaders).

Our second contribution relates to the study of institutional flexibility. For this purpose, we enrich the above-mentioned framework with shocks that change the competences of different types of governments (thus capturing potential changes in the needs of the society for different types of skills and expertise). Although the systematic and tractable analysis of this class of dynamic games is challenging, we provide a characterization of the structure of equilibria when stochastic shocks are sufficiently infrequent. Using this characterization, we show how the quality (competence level) of governments evolves in the presence of stochastic shocks and how this evolution is impacted by political institutions. While, without shocks, a greater degree of democracy (beyond perfect democracy) does not necessarily guarantee a better government, the pattern that emerges when we turn to institutional flexibility is different. In particular, our analysis shows that a greater degree of democracy leads to better outcomes in the long run; in particular, it increases the probability that the best government will be in power. Intuitively, this is because a greater degree of democracy enables greater adaptability to changes in the environment (which alter the relative ranking of governments in terms of quality). This result therefore formalizes the notion that more democratic institutions ensure greater flexibility.[4] At a slightly more technical level, this result reflects the fact that when the degree of democracy is high, there are "relatively few" other stable governments near a stable government, so a shock that destabilizes the current government likely leads to a big jump in competence.

Finally, we also show that in the presence of shocks, "royalty-like" nondemocratic regimes, where some individuals must always be in the government, may lead to better long-run outcomes than "junta-like" regimes, where a subset of the current members of the junta can block change (even though no specific member is essential). The royalty-like regimes might sometimes allow greater adaptation to change because one (or more) of the members of the initial government is secure in her position. In contrast, as discussed above, without such security the fear of further changes might block all competence-increasing reforms in government.

We now illustrate some of the basic ideas with a simple example.

[4] The stochastic analysis also shows that random shocks to the identity of the members of the government may sometimes lead to better governments in the long run because they destroy stable incompetent governments. Besley (2005) writes: "History suggests that four main methods of selection to political office are available: drawing lots, heredity, the use of force and voting." Our model suggests why, somewhat paradoxically, drawing lots, which was used in Ancient Greece, might sometimes lead to better long-run outcomes than the alternatives.

Example 1 Suppose that the society consists of $n \geq 6$ of individuals. Assume that any $k = 3$ individuals could form a government. A change in government requires both the support of the majority of the population and the consent of $l = 1$ member of the government, so that there is an imperfect democracy, with a "minimal" degree of incumbency advantage. Suppose that individual j has a level of competence γ_j, and order the individuals, without loss of any generality, in descending order according to their competence, so $\gamma_1 > \gamma_2 > ... > \gamma_n$. The competence of a government is the sum of the competences of its three members. Each individual obtains utility from the competence level of the government and also a large rent from being in office, so that each prefers to be in office regardless of the competence level of the government. Suppose also that individuals have a sufficiently high discount factor, so that the future matters relative to the present.

It is straightforward to determine the stable governments that will persist and remain in power once formed. Evidently, $\{1, 2, 3\}$ is a stable government, since it has the highest level of competence, so neither a majority of outsiders nor members of the government would like to initiate a change (some outsiders may want to initiate a change: for example, 4, 5, and 6 would prefer government $\{4, 5, 6\}$, but they do not have the power to enforce such a change). In contrast, governments of the form $\{1, i, j\}$, $\{i, 2, j\}$, and $\{i, j, 3\}$ are unstable (for $i, j > 3$), which means that starting with these governments, there will necessarily be a change. In particular, in each of these cases, $\{1, 2, 3\}$ will receive support from both one current member of government and from the rest of the population, who would be willing to see a more competent government.

Consider next the case where $n = 6$ and suppose that the society starts with the government $\{4, 5, 6\}$. This is also a stable government, even though it is the lowest competence government and thus the worst possible option for the society as a whole. This is because any change in government must result in a new government of one of the following three forms: $\{1, i, j\}$, $\{i, 2, j\}$, or $\{i, j, 3\}$. But we know that all of these types of governments are unstable. Therefore, any of the more competent governments will ultimately take us to $\{1, 2, 3\}$, which does not include any of the members of the initial government. Since individuals are relatively patient, none of the initial members of the government would support (consent to) a change that will ultimately exclude them. As a consequence, the initial worst government persists forever. Returning to our discussion of the unwillingness of certain governments to include skilled technocrats, this example shows why such a technocrat, for example individual 1, will not be included in the government $\{4, 5, 6\}$, even though he would potentially increase the quality and competence of the government substantially.

One can also verify easily that $\{4, 5, 6\}$ is also a stable government when $l = 3$, since in this case any change requires the support of all three members of government and none of them would consent to a change. Therefore, a greater degree of democracy (lower l) does not guarantee better outcomes in the long run. In contrast, under $l = 2$, $\{4, 5, 6\}$ is not a stable government, so the lower degree of democracy can improve the quality of the government.

Now consider the same environment as above, but with potential changes in the competences of the agents. For example, individual 4 may see an increase in his competence, so that he becomes the third most competent agent (i.e., $\gamma_4' \in (\gamma_3, \gamma_2)$). Suppose that shocks are sufficiently infrequent so that stability of governments in periods without shocks is given by the same reasoning as for the nonstochastic case. Consider the situation starting with the government $\{4, 5, 6\}$ and $l = 1$. Then, this government will remain in power until the shock occurs. Nevertheless, the equilibrium government will eventually converge to $\{1, 2, 3\}$; at some point a shock will change the relative competences of agents 3 and 4, and the government $\{4, 5, 6\}$ would become unstable, because now individual 4 would support the emergence of the government $\{1, 2, 4\}$, which now has the highest competence. In contrast, when $l = 3$, the ruling government will remain in power even after the shock. This simple example thus illustrates how, even though a greater degree of democracy does not ensure better outcomes in nonstochastic environments, it may provide greater flexibility and hence better long-run outcomes in the presence of shocks.

Our paper is related to several different literatures. While much of the literature on political economy focuses on the role of political institutions in providing (or failing to provide) the right incentives to politicians (see, among others, Barro, 1973, Ferejohn, 1986, Besley and Case, 1995, Persson, Roland and Tabellini, 1997, Niskanen, 1971, Shleifer and Vishny, 1993, Acemoglu, Robinson and Verdier, 2004, Padro-i-Miquel, 2007), there is also a small (but growing) literature investigating the selection of politicians, most notably, Banks and Sundaram (1998), Diermeier, Keane, and Merlo (2005), and Besley (2005). The main challenge facing the society and the design of political institutions in these papers is that the ability and motivations of politicians are not observed by voters or outside parties. While such information-related selection issues are undoubtedly important, our paper focuses on the difficulties in ensuring that the "right" government is selected even when information is perfect and common. Also differently from these literatures, we emphasize the importance of institutional flexibility in the face of shocks.

Besley and Coate (1997, 1998), Osborne and Slivinski (1996), Bueno de Mesquita et al. (2003), Caselli and Morelli (2004), Messner and Polborn (2004), Mattozzi and Merlo (2006), and Besley and Kudamatsu (2009) provide alternative and complementary "theories of bad governments/politicians". For example, Bueno de Mesquita et al. (2003) emphasize the composition of the "selectorate," the group of players that can select governments, as an important factor in leading to inefficient governments and policies. Caselli and Morelli (2004) suggest that voters might be unwilling to replace the corrupt incumbent by a challenger whom they expect to be equally corrupt. Mattozzi and Merlo (2006) argue that more competent politicians have higher opportunity costs of entering politics. Also notable is the recent work by Besley and Kudamatsu (2009), which relates the success of autocratic regimes to their ability to select politicians, which is in turn related to the composition of the "selectorate" (as defined by Bueno de Mesquita et al., 2003). However, these papers do not develop the potential persistence in bad governments resulting from dynamics of government formation and do not focus on the importance of institutional flexibility. We are also not aware of other papers providing a comparison of different political regimes in terms of the selection of politicians under nonstochastic and stochastic conditions.

McKelvey and Reizman (1992) suggest that seniority rules in the Senate and the House create an endogenous incumbency advantage, and current members of these bodies will have an incentive to introduce such seniority rules. Our results are also related to recent work on the persistence of bad governments and inefficient institutions, including Acemoglu and Robinson (2008), Acemoglu, Ticchi, and Vindigni (2007), and Egorov and Sonin (2004).[5]

More closely related to our work are prior analyses of dynamic political equilibria in the context of club formation as in Roberts (1991) and Barbera, Maschler, and Shalev (2001), as well as dynamic analyses of choice of constitutions and equilibrium political institutions as in Acemoglu and Robinson (2006), Barbera and Jackson (2004), Matthias and Polborn (2004), and Lagunoff (2006). Our recent work, Acemoglu, Egorov, and Sonin (2008), provides a general framework for the analysis of the dynamics of constitutions, coalitions and clubs. The current paper is a continuation of this line of research. It differs from our previous work in a number of important dimensions. First, the focus here is on the substantive questions concerning the relationship between different political institutions and the selection of politicians and governments, which is new, relatively unexplored, and (in our view) important. Second, this paper extends our previous work by allowing for stochastic shocks and enables us to investigate issues of institutional flexibility. Third, it involves for a structure of preferences for which our previous results cannot be directly applied.[6]

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 introduces the concept of (Markov) political equilibrium, which allows a general and tractable characterization of equilibria in this class of games. Section 4 provides our main results on the comparison of different regimes in terms of selection of governments and politicians. Section 5 extends the analysis to allow for stochastic changes in the competences of the members of the society and presents a comparison of different regimes in the presence of stochastic shocks.

[5] Acemoglu (2008) also emphasizes the potential benefits of democracy in the long run, but through a different channel—because the alternative, oligarchy, creates entry barriers and sclerosis.

[6] In particular, the results in Acemoglu, Egorov and Sonin (2008) apply under a set of acyclicity conditions. Such acyclicity does not hold in the current paper (see Appendix B). This makes the general characterization of the structure of equilibria both more challenging and of some methodological interest.

Section 6 concludes. Appendix A contains all of the proofs of the results stated in the text, while Appendix B considers an extensive-form game with explicitly specified proposal and voting procedures, and shows the equivalence between the Markov perfect equilibria of this game and the (simpler) notion of political equilibrium used in the text.

2 Model

We consider a dynamic game in discrete time indexed by $t = 0, 1, 2, \ldots$. The population is represented by the set $\mathcal{I}$ and consists of $n < \infty$ individuals. We refer to non-empty subsets of $\mathcal{I}$ as *coalitions* and denote the set of coalitions by $\mathcal{C}$. We also designate a subset of coalitions $\mathcal{G} \subset \mathcal{C}$ as the set of *feasible governments*. For example, the set of feasible governments could consist of all groups of individuals of size k_0 (for some integer k_0) or all groups of individuals of size greater than k_1 and less than some other integer k_2. To simplify the discussion, we define $\bar{k} = \max_{G \in \mathcal{G}} |G|$, so $\bar{k}$ is the upper bound for the size of any feasible government: i.e., for any $G \in \mathcal{G}$, $|G| \leq \bar{k}$. It is natural to presume that $\bar{k} < n/2$.

In each period, the society is ruled by one of the feasible governments $G^t \in \mathcal{G}$. The initial government G^0 is given as part of the description of the game and G^t for $t > 0$ is determined in equilibrium as a result of the political process described below. The government in power at any date affects three aspects of the society:

1. It influences collective utilities (for example, by providing public goods or influencing how competently the government functions).

2. It determines individual utilities (members of the government may receive additional utility because of rents of being in office or corruption).

3. It indirectly influences the future evolution of governments by shaping the distribution of political power in the society (for example, by creating incumbency advantage in democracies or providing greater decision-making power or veto rights to members of the government under alternative political institutions).

We now describe each of these in turn. The influence of the government on collective utilities is modeled via its *competence*. In particular, at each date t, there exists a function

$$\Gamma^t : \mathcal{G} \rightarrow \mathbb{R}$$

designating the competence of each feasible government $G \in \mathcal{G}$ (at that date). We refer to $\Gamma^t_G \in \mathbb{R}$ as government G's competence, with the convention that higher values correspond to greater competence. In Section 4, we will assume that each individual has a certain level of competence or ability, and the competence of a government is a function of the abilities of its members. For now, this additional assumption is not necessary. Note also that the function Γ^t depends on time. This generality is introduced to allow for changes in the environment (in particular, changes in the relative competences of different individuals and governments).

Individual utilities are determined by the competence of the government that is in power at that date and by whether the individual in question is herself in the government. More specifically, each individual $i \in \mathcal{I}$ at time τ has discounted (expected) utility given by

$$U_i^\tau = \mathbb{E} \sum\nolimits_{t=\tau}^{\infty} \beta^{(t-\tau)} u_i^t, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor and u_i^t is individual's stage payoff, given by

$$u_i^t = w_i \left(G^t, \Gamma_{G^t}^t\right) = w_i \left(G^t\right), \tag{2}$$

where in the second equality we suppress dependence on $\Gamma_{G^t}^t$ to simplify notation; we will do this throughout unless special emphasis is necessary. Throughout, we impose the following assumptions on w_i.

Assumption 1 *The function w_i satisfies the following properties:*

1. *for each $i \in \mathcal{I}$ and any $G, H \in \mathcal{G}$ such that $\Gamma_G^t > \Gamma_H^t$: if $i \in G$ or $i \notin H$, then $w_i(G) > w_i(H)$.*

2. *for any $G, H \in \mathcal{G}$ and any $i \in G \setminus H$, $w_i(G) > w_i(H)$.*

Part 1 of this assumption is a relatively mild restriction on payoffs. It implies that all else equal, more competent governments give higher stage payoff. In particular, if an individual belongs to both governments G and H, and G is more competent than H, then she prefers G. The same conclusion also holds when the individual is not a member of either of these two governments or when she is only a member of G (and not of H). Therefore, this part of the assumption implies that the only situation in which an individual may prefer a less competent government to a more competent one is when she is a member of the former, but not of the latter. This simply captures the presence of rents from holding office or additional income from being in government due to higher salaries or corruption. The interesting interactions in our setup result from the "conflict of interest": individuals prefer to be in the government even when this does not benefit the rest of the society. Part 2 of the assumption strengthens the first part and imposes that this conflict of interest is always present; that is, individuals receive higher payoffs from governments that include them than from those that exclude them (regardless of the competence levels of the two governments). We impose both parts of this assumption throughout. It is important to note that Assumption 1 implies that all voters, who are not part of the government, care about a one-dimensional government competence; this feature simplifies the analysis considerably. Nevertheless, the tractability of our framework makes it possible to enrich this environment by allowing other sources of disagreement or conflict of interest among voters, and we return to this issue in the Conclusion.

We next provide an example that makes some of these notions slightly more concrete.

Example 2 Suppose that the competence of government G, Γ_G, is the amount of public good produced in the economy under feasible government G, and

$$w_i(G) = v_i(\Gamma_G) + b_i \mathbf{I}_{\{i \in G\}}, \tag{3}$$

where $v_i : \mathbb{R} \to \mathbb{R}$ is a strictly increasing function (for each $i \in \mathcal{I}$) corresponding to the utility from public good for individual i, b_i is a measure of the rents that individual i obtains from being in office, and $\mathbf{I}_X$ is the indicator of event X. If $b_i \geq 0$ for each $i \in \mathcal{I}$, then (3) satisfies part 1 of Assumption 1. In addition, if b_i is sufficiently large for each i, then each individual prefers to be a member of the government, even if this government has a very low level of competence, thus part 2 of Assumption 1 is also satisfied.

Finally, the government in power influences the determination of future governments whenever consent of some current government members is necessary for change. We represent the set of individuals (regular citizens and government members) who can, collectively, induce a change in government by specifying the set of *winning coalitions*, $\mathcal{W}_G$, which is a function of current government G (for each $G \in \mathcal{G}$). This is an economical way of summarizing the relevant information, since the set of winning coalitions is precisely the subsets of the society that are able to force (or to block) a change in government. We only impose a minimal amount of structure on the set of winning coalitions.

Assumption 2 *For any feasible government $G \in \mathcal{G}$, $\mathcal{W}_G$ is given by*

$$\mathcal{W}_G = \{X \in \mathcal{C} : |X| \geq m_G \text{ and } |X \cap G| \geq l_G\},$$

where l_G and m_G are integers satisfying $0 \leq l_G \leq |G| \leq \bar{k} < m_G \leq n - \bar{k}$ (recall that $\bar{k}$ is the maximal size of the government and n is the size of the society).

The restrictions imposed in Assumption 2 are intuitive. In particular, they state that a new government can be instituted if it receives a sufficient number of votes from the entire society (m_G total votes) *and* if it receives support from some subset of the members of the current government (l_G of the current government members need to support such a change). This definition allows l_G to be any number between 0 and $|G|$. One special feature of Assumption 2 is that it does not relate the number of veto players in the current government, l_G, to the total number of individuals in the society who wish to change the government, m_G. This aspect of Assumption 2 can be relaxed without affecting our general characterization; we return to a discussion of this issue in the Conclusion.

Given this notation, the case where $l_G = 0$ should be thought of as *perfect democracy*, where current members of the government have no special power, and the case where $l_G = |G|$ as *extreme dictatorship*, where unanimity among government members is necessary for any change. Between these extremes are imperfect democracies (or less strict forms of dictatorships), which may arise either because there is some form of (strong or weak) incumbency advantage in democracy or because current government (junta) members are able to block the introduction of a new government. Note also that we imposed some mild assumptions on m_G. In particular, less than $\bar{k}$ individuals is insufficient for a change to take place. This ensures that a rival government cannot take power without any support from other individuals (recall that $\bar{k}$ denotes the maximum size of the government, so the rival government must have no more than $\bar{k}$ members), and $m_G \leq n - \bar{k}$ individuals are sufficient to implement a change provided that l_G members of the current government are among them. For example, these requirements are naturally met when $\bar{k} < n/2$ and $m_G = \lfloor (n+1)/2 \rfloor$ (i.e., majority rule).[7]

In addition to Assumptions 1 and 2, we also impose the following *genericity* assumption, which ensures that different governments have different competences. This assumption simplifies the notation and is without much loss of generality, since if it were not satisfied for a society, any small perturbation of competence levels would restore it.

Assumption 3 *For any $t \geq 0$ and any $G, H \in \mathcal{G}$ such that $G \neq H$, $\Gamma_G^t \neq \Gamma_H^t$.*

3 Political Equilibria in Nonstochastic Environments

In this section, we focus on nonstochastic environments, where $\Gamma^t = \Gamma$ (or $\Gamma_G^t = \Gamma_G$ for all $G \in \mathcal{G}$). For these environments, we introduce our equilibrium concept, *(Markov) political equilibrium*, and show that equilibria have a simple recursive characterization.[8] We return to the more general stochastic environments in Section 5.

[7] Recall also that $\lfloor x \rfloor$ denotes the integer part of a real number x.

[8] Throughout, we refer to this equilibrium concept as "political equilibrium" or simply as "equilibrium". We do not use the acronym MPE, which will be used for Markov perfect equilibrium in Appendix B.

3.1 Political Equilibrium

Our equilibrium concept, (Markov) political equilibrium, imposes that only transitions from the current government to a new government that maximize the discounted utility of a winning coalition will take place; and if no such transition exists, the current government will be *stable* (i.e., it will persist in equilibrium). The qualifier "Markov" is added since this definition implicitly imposes that transitions from the current to a new government depend on the current government—not on the entire history.

To introduce this equilibrium concept more formally, let us first define the *transition rule* $\phi : \mathcal{G} \to \mathcal{G}$, which maps each feasible government G in power at time t to the government that would emerge in period $t+1$.[9] Given ϕ, we can write the discounted utility implied by (1) for each individual $i \in \mathcal{I}$ starting from current government $G \in \mathcal{G}$ recursively as $V_i(G \mid \phi)$, given by

$$V_i(G \mid \phi) = w_i(G) + \beta V_i(\phi(G) \mid \phi) \text{ for all } G \in \mathcal{G}. \tag{4}$$

Intuitively, starting from $G \in \mathcal{G}$, individual $i \in \mathcal{I}$ receives a current payoff of $w_i(G)$. Then ϕ (uniquely) determines next period's government $\phi(G)$, and thus the continuation value of this individual, discounted to the current period, is $\beta V_i(\phi(G) \mid \phi)$.

A government G is *stable* given mapping ϕ if $\phi(G) = G$. In addition, we say that ϕ is *acyclic* if for any (possibly infinite) chain $H_1, H_2, \ldots \subset \mathcal{G}$ such that $H_{k+1} \in \phi(H_k)$, and any $a < b < c$, if $H_a = H_c$ then $H_a = H_b = H_c$.

Given (4), the next definition introduces the notion of a political equilibrium, which will be represented by the mapping ϕ provided that two conditions are met.

Definition 1 *A mapping $\phi : \mathcal{G} \to \mathcal{G}$ constitutes a* (Markov) political equilibrium *if for any $G \in \mathcal{G}$, the following two conditions are satisfied:*

(i) either the set of players who prefer $\phi(G)$ to G (in terms of discounted utility) forms a winning coalition, i.e., $S = \{i \in \mathcal{I} : V_i(\phi(G) \mid \phi) > V_i(G \mid \phi)\} \in \mathcal{W}_G$, (or equivalently $|S| \geq m_G$ and $|S \cap G| \geq l_G$); or else, $\phi(G) = G$;

(ii) there is no alternative government $H \in \mathcal{G}$ that is preferred both to a transition to $\phi(G)$ and to staying in G permanently, i.e., there is no H such that $S'_H = \{i \in \mathcal{I} : V_i(H \mid \phi) > V_i(\phi(G) \mid \phi)\} \in \mathcal{W}_G$ and $S''_H = \{i \in \mathcal{I} : V_i(H \mid \phi) > w_i(G)/(1-\beta)\} \in \mathcal{W}_G$ (alternatively, for any alternative H, either $|S'_H| < m_G$, or $|S'_H \cap G| < l_G$, or $|S''_H| < m_G$, or $|S''_H \cap G| < l_G$).

[9] In principle, ϕ could be set-valued, mapping from $\mathcal{G}$ into $\mathcal{P}(\mathcal{G})$ (the power set of $\mathcal{G}$), but our analysis below shows that, thanks to Assumption 3, its image is always a singleton (i.e., it is a "function" rather than a "correspondence," and also by implication, it is uniquely defined). We impose this feature to simplify the notation.

This definition states that a mapping ϕ constitutes a political equilibrium ("is a political equilibrium") if it maps the current government G to alternative $\phi(G)$ that (unless it coincides with G) must be preferred to G by a sufficient majority of the population and a sufficient number of current government members (so as not to be blocked). Note that in part (i), the set S could have alternatively been written as $S = \{i \in \mathcal{I} : V_i(\phi(G) \mid \phi) > w_i(G) / (1-\beta)\}$, since if this set is not a winning coalition, then $\phi(G) = G$ and thus $V_i(G \mid \phi) = w_i(G) / (1-\beta)$. Part (ii) of the definition requires that there does not exist another alternative H that would have been a "more preferable" transition; that is, there should be no H that is preferred both to a transition to $\phi(G)$ and to staying in G forever by a sufficient majority of the population and a sufficient number of current government members. The latter condition is imposed, since if there exists a subset H that is preferred to a transition to $\phi(G)$ but not to staying in G forever, then at each stage a move to H can be blocked.[10]

We take the definition of political equilibrium given in Definition 1 as a primitive and use it in this and the next section. The advantage of this definition is that it is simple and economical. A possible disadvantage is that it does not explicitly specify how offers for different types of transitions are made and the exact sequences of events at each stage.[11] In Appendix B, we specify the sequences in which offers are made, voting takes place, and when transitions can take place explicitly, and characterize the Markov perfect equilibria (MPE) of this extended environment. We show that for high discount factors, MPE coincide with our definition of political equilibria.

3.2 General Characterization

We now prove the existence and provide a characterization of political equilibria. We start with a recursive characterization of the mapping ϕ described in Definition 1. Let us enumerate the elements of the set $\mathcal{G}$ as $\{G_1, G_2, ..., G_{|\mathcal{G}|}\}$ such that $\Gamma_{G_x} > \Gamma_{G_y}$ whenever $x < y$. With this enumeration, G_1 is the most competent ("best") government, while $G_{|\mathcal{G}|}$ is the least competent government. In view of Assumption 3, this enumeration is well defined and unique.

Now, suppose that for some $q > 1$, we have defined ϕ for all G_j with $j < q$. Define the set

$$\mathcal{M}_q \equiv \{j : 1 \leq j < q,\ \{i \in \mathcal{I} : w_i(G_j) > w_i(G_q)\} \in \mathcal{W}_{G_q}, \text{ and } \phi(G_j) = G_j\}. \tag{5}$$

Note that this set depends simply on stage payoffs in (2), *not* on the discounted utilities defined in (4), which are "endogenous" objects. This set can thus be computed easily from the primitives

[10] The explicit game form in Appendix B clarifies this further.

[11] In this regard, our equilibrium concept is similar to the concept of Markov voting equilibrium in Roberts (1999).

of the model (for each q). Given this set, let the mapping ϕ be

$$\phi(G_q) = \begin{cases} G_q & \text{if } \mathcal{M}_q = \varnothing; \\ G_{\min\{j \in \mathcal{M}_q\}} & \text{if } \mathcal{M}_q \neq \varnothing. \end{cases} \tag{6}$$

Since the set $\mathcal{M}_q$ is well defined, the mapping ϕ is also well defined, and by construction it is single valued. Theorems 1 and 2 next show that, for sufficiently high discount factors, this mapping constitutes the unique acyclic political equilibrium and that, under additional mild conditions, it is also the unique political equilibrium (even considering possible cyclic equilibria).

Theorem 1 *Suppose that Assumptions 1-3 hold and let $\phi : \mathcal{G} \to \mathcal{G}$ be as defined in* (6). *Then there exists $\beta_0 < 1$ such that for any discount factor $\beta > \beta_0$, ϕ is the unique acyclic political equilibrium.*

Proof. See Appendix A. ■

Let us now illustrate the intuition for why the mapping ϕ constitutes a political equilibrium. Recall that G_1 is the most competent ("best") government. It is clear that we must have $\phi(G_1) = G_1$, since all members of the population that are not in G_1 will prefer it to any other $G' \in \mathcal{G}$ (from Assumption 1). Assumption 2 then ensures that there will not be a winning coalition in favor of a permanent move to G'. However, G' may not persist itself, and it may eventually lead to some alternative government $G'' \in \mathcal{G}$. But in this case, we can apply this reasoning to G'' instead of G', and thus the conclusion $\phi(G_1) = G_1$ applies. Next suppose we start with government G_2 in power. The same argument applies if G' is any one of $G_3, G_4, \ldots, G_{|\mathcal{G}|}$. One of these may eventually lead to G_1, thus for sufficiently high discount factors, a sufficient majority of the population may support a transition to such a G' in order to eventually reach G_1. However, discounting also implies that in this case, a sufficient majority would also prefer a direct transition to G_1 to this dynamic path (recall part (ii) of Definition 1). So the relevant choice for the society is between G_1 and G_2. In this comparison, G_1 will be preferred if it has sufficiently many supporters, that is, if the set of individuals preferring G_1 to G_2 is a winning coalition within G_2, or more formally if

$$\{i \in \mathcal{I} : w_i(G_1) > w_i(G_2)\} \in \mathcal{W}_{G_2}.$$

If this is the case, $\phi(G_2) = G_1$; otherwise, $\phi(G_2) = G_2$. This is exactly what the function ϕ defined in (6) stipulates. Now let us start from government G_3. We then only need to consider the choice between G_1, G_2, and G_3. To move to G_1, it suffices that a winning coalition within G_3 prefers G_1 to G_3.[12] However, whether the society will transition to G_2 depends on the stability

[12] If some winning coalition also prefers G_2 to G_3, then G_1 should still be chosen over G_2, because only members of G_2 who do not belong to G_1 prefer G_2 to G_1, and Assumption 2 ensures that those preferring G_1 over G_2 (starting in G_3) also form a winning coalition. Then a transition to G_2 is ruled out by part (ii) of Definition 1.

of G_2. In particular, we may have a situation in which G_2 is not a stable government, which, by necessity, implies that $\phi(G_2) = G_1$. Then a transition to G_2 will lead to a permanent transition to G_1 in the next period. However, this sequence may be non-desirable for some of those who prefer to move to G_2. In particular, there may exist a winning coalition in G_3 that prefers to stay in G_3 rather than transitioning permanently to G_1 (and as a consequence, there is no winning coalition that prefers such a transition), even though there also exists a winning coalition in G_3 that would have preferred a permanent move to G_2. Writing this more explicitly, we may have that

$$\{i \in \mathcal{I} : w_i(G_2) > w_i(G_3)\} \in \mathcal{W}_{G_3},$$

but

$$\{i \in \mathcal{I} : w_i(G_1) > w_i(G_3)\} \notin \mathcal{W}_{G_3}.$$

If so, the transition from G_3 to G_2 may be blocked with the anticipation that it will lead to G_1 which does not receive the support of a winning coalition within G_3. This reasoning illustrates that for a transition to take place, not only should the target government be preferred to the current one by a winning coalition (starting from the current government), but also that the target government should be "stable," i.e., $\phi(G') = G'$. This is exactly the requirement in (6). In this light, the intuition for the mapping ϕ and thus for Theorem 1 is that a government G will persist in equilibrium (will be stable) if there does not exist another stable government receiving support from a winning coalition (a sufficient majority of the population and the required number of current members of government).

Theorem 1 states that ϕ in (6) is the unique acyclic political equilibrium. However, it does not rule out cyclic equilibria. We provide an example of a cyclic equilibrium in Appendix B (see Example 11). Cyclic equilibria are unintuitive and "fragile". We next show that they can also be ruled out under a variety of relatively weak assumptions. The next theorem thus strengthens Theorem 1 so that ϕ in (6) is the unique political equilibrium (among both cyclic and acyclic ones).

Theorem 2 *The mapping ϕ defined in (6) is the unique political equilibrium (and hence in the light of Theorem 1, any political equilibrium is acyclic) if any of the following conditions holds:*

1. *For any $G \in \mathcal{G}$, $|G| = k$, $l_G = l$ and $m_G = m$ for some k, l and m.*

2. *For any $G \in \mathcal{G}$, $l_G \geq 1$.*

3. *For any collection of different feasible governments $H_1, \ldots, H_q \in \mathcal{G}$ (for $q \geq 2$) and for all $i \in \mathcal{I}$, we have $w_i(H_1) \neq \left(\sum_{p=1}^{q} w_i(H_p)\right)/q$.*

4. $\theta > \varepsilon \cdot |\mathcal{G}|$, *where* $\theta \equiv \min_{\{i \in \mathcal{I} \text{ and } G, H \in \mathcal{G}:\ i \in G \setminus H\}} \{w_i(G) - w_i(H)\}$ *and* $\varepsilon \equiv \max_{\{i \in \mathcal{I} \text{ and } G, H \in \mathcal{G}:\ i \in G \cap H\}} \{w_i(G) - w_i(H)\}$.

Proof. See Appendix A. ■

This theorem states four relatively mild conditions under which there are no cyclic equilibria (thus making ϕ in (6) the unique equilibrium). First, if all feasible governments have the same size, k, the same degree of incumbency advantage, l, and the same threshold for the required number of total votes for change, m, then all equilibria must be acyclic and thus ϕ in (6) is the unique political equilibrium. Second, the same conclusion applies if we always need the consent of at least one member of the current government for a transition to a new government. These two results imply that cyclic equilibria are only possible if starting from some governments, there is no incumbency advantage and either the degree of incumbency advantage or the vote threshold differs across governments (see Example 11 in Appendix B). The third part of the theorem proves that there are no acyclic political equilibria under a very mild assumption. In fact, the requirement on payoffs imposed in this part of the theorem is a slight strengthening of Assumption 3 and also holds generically (meaning that if it did not hold, a small perturbation of payoff functions would restore it).[13] Finally, the fourth part of the theorem provides a condition on preferences that also rules out cyclic equilibria. In particular, this condition states that if each individual receives sufficiently high utility from being in government (greater than θ) and does not care much about the composition of the rest of the government (the difference in her utility between any two governments including her is always less than ε), then all equilibria must be acyclic. In Appendix B, we show (Example 11) how a cyclic political equilibrium is possible if neither of the four sufficient conditions in Theorem 2 holds.

4 Characterization of Nonstochastic Transitions

In this section, we compare different political regimes in terms of their ability to select governments with high levels of competence. To simplify the exposition and focus on the more important interactions, we assume that all feasible governments have the same size, $k \in \mathbb{N}$, where $k < n/2$. More formally, let us define

$$\mathcal{C}^k = \{Y \in \mathcal{C} : |Y| = k\}.$$

Then, $\mathcal{G} = \mathcal{C}^k$. In addition, we assume that for any $G \in \mathcal{G}$, $l_G = l \in \mathbb{N}$ and $m_G = m \in \mathbb{N}$, so that the set of winning coalitions can be simply expressed as

$$\mathcal{W}_G = \{X \in \mathcal{C} : |X| \geq m \text{ and } |X \cap G| \geq l\}, \tag{7}$$

[13]This requirement is exactly the same as Assumption 3′ we impose in Appendix B in the analysis of the extensive form game.

where $0 \leq l \leq k < m \leq n-k$. This specification implies that given n, k, and m, the number l corresponds to an inverse measure of democracy. If $l=0$, then all individuals have equal weight and there is no incumbency advantage, thus we have a *perfect democracy*. In contrast, if $l>0$, the consent of some of the members of the government is necessary for a change, thus we have an *imperfect democracy*. We thus have strengthened Assumption 2 to the following.

Assumption 2$'$ *We have that $\mathcal{G} = \mathcal{C}^k$, and that there exist integers l and m such that the set of winning coalitions is given by (7).*

In view of part 1 of Theorem 2, Assumption 2$'$ ensures that the acyclic political equilibrium ϕ given by (6) is the unique equilibrium; naturally, we will focus on this equilibrium throughout the rest of the analysis. In addition, given this additional structure, the mapping ϕ can be written in a simpler form. Recall that governments are still ranked according to their level of competence, so that G_1 denotes the most competent government. Then we have:

$$\mathcal{M}_q = \{j : 1 \leq j < q,\ |G_j \cap G_q| \geq l, \text{ and } \phi(G_j) = G_j\}, \tag{8}$$

and, as before,

$$\phi(G_q) = \begin{cases} G_q & \text{if } \mathcal{M}_q = \varnothing; \\ G_{\min\{j \in \mathcal{M}_q\}} & \text{if } \mathcal{M}_q \neq \varnothing. \end{cases} \tag{9}$$

Naturally, the mapping ϕ is again well defined and unique. Finally, let us also define

$$\mathcal{D} = \{G \in \mathcal{G} : \phi(G) = G\}$$

as the set of stable governments (the fixed points of mapping ϕ). If $G \in \mathcal{D}$, then $\phi(G) = G$, and this government will persist forever if it is the initial government of the society.

We now investigate the structure of stable governments and how it changes as a function of the underlying political institutions, in particular, the degree of democracy. Throughout this section, we assume that Assumptions 1, 2$'$ and 3 hold, and we do not add these qualifiers to any of the propositions to economize on space.

Our first proposition provides an important technical result (part 1). It then uses this result to show that perfect democracy ensures the emergence of the best (most competent) government, but any departure from perfect democracy destroys this result and enables the emergence of highly incompetent/inefficient governments. It also shows that extreme dictatorship makes all initial governments stable, regardless of how low their competences may be.

Proposition 1 *The set of stable feasible governments $\mathcal{D}$ satisfies the following properties.*

1. *If $G, H \in \mathcal{D}$ and $|G \cap H| \geq l$, then $G = H$. In other words, any two distinct stable governments may have at most $l-1$ common members.*

2. *Suppose that* $l = 0$, *so that the society is a* perfect democracy. *Then* $\mathcal{D} = \{G_1\}$. *In other words, starting from any initial government, the society will transition to the most competent government.*

3. *Suppose* $l \geq 1$, *so that the society is an* imperfect democracy *or a* dictatorship. *Then there are at least two stable governments, i.e.,* $|\mathcal{D}| \geq 2$. *Moreover, the least competent governments may be stable.*

4. *Suppose* $l = k$, *so that the society is an* extreme dictatorship. *Then* $\mathcal{D} = \mathcal{G}$, *so any feasible government is stable.*

Proof. See Appendix A. ■

Proposition 1 shows the fundamental contrast between perfect democracy, where there is no incumbency advantage, and other political institutions, which provide some additional power to "insiders" (current members of the government). With perfect democracy, the best government will necessarily emerge. With any deviation from perfect democracy, there will necessarily exist at least one other stable government (by definition less competent than the best), and even the worst government might be stable. The next example supplements Example 1 from the Introduction by showing a richer environment in which the least competent government is stable.

Example 3 Suppose $n = 9$, $k = 3$, $l = 1$, and $m = 5$, so that a change in government requires support from a simple majority of the society, including at least one member of the current government. Suppose $\mathcal{I} = \{1, 2, \ldots, 9\}$, and that stage payoffs are given by (3) in Example 2. Assume also that $\Gamma_{\{i_1,i_2,i_3\}} = 1000 - 100i_1 - 10i_2 - i_3$ (for $i_1 < i_2 < i_3$). This implies that $\{1, 2, 3\}$ is the most competent government, and is therefore stable. Any other government that includes 1 or 2 or 3 is unstable. For example, the government $\{2, 5, 9\}$ will transit to $\{1, 2, 3\}$, as all individuals except 5 and 9 prefer the latter. However, government $\{4, 5, 6\}$ is stable: any government that is more competent must include 1 or 2 or 3, and therefore is either $\{1, 2, 3\}$ or will immediately transit to $\{1, 2, 3\}$, which means that any such transition will not receive support from any of the members of $\{4, 5, 6\}$. Now, proceeding inductively, we find that any government other than $\{1, 2, 3\}$ and $\{4, 5, 6\}$ that contains at least one individual $1, 2, \ldots, 6$ is unstable. Consequently, government $\{7, 8, 9\}$, which is the least competent government, is stable.

Proposition 1 establishes that with any regime other than perfect democracy, there will necessarily exist stable inefficient/incompetent governments and these may in fact have quite low levels of competences. It does not, however, provide a characterization of when highly incompetent governments will be stable.

We next provide a systematic answer to this question focusing on societies with large numbers of individuals (i.e., n large). Before doing so, we introduce an assumption that will be used in the third part of the next proposition and in later results. In particular, in what follows we will sometimes suppose that each individual $i \in \mathcal{I}$ has a level of ability (or competence) given by $\gamma_i \in \mathbb{R}_+$ and that the competence of the government is a strictly increasing function of the abilities of its members. This is more formally stated in the next assumption.

Assumption 4 *Suppose $G \in \mathcal{G}$, and individuals $i, j \in \mathcal{I}$ are such that $i \in G$, $j \notin G$, and $\gamma_i \geq \gamma_j$. Then $\Gamma_G \geq \Gamma_{(G \setminus \{i\}) \cup \{j\}}$.*

The canonical form of the competence function consistent with Assumption 4 is

$$\Gamma_G = \sum\nolimits_{i \in G} \gamma_i, \tag{10}$$

though for most of our analysis, we do not need to impose this specific functional form.

Assumption 4 is useful because it enables us to rank individuals in terms of their "abilities". This ranking is strict, since Assumptions 3 and 4 together imply that $\gamma_i \neq \gamma_j$ whenever $i \neq j$. When we impose Assumption 4, we also enumerate individuals according to their abilities, so that $\gamma_i > \gamma_j$ whenever $i < j$.

The next proposition shows that for societies above a certain threshold of size (as a function of k and l), there always exist stable governments that contain no member of the ideal government and no member of any group of certain prespecified sizes (thus, no member of groups that would generate a range of potentially high competence governments). Then, under Assumption 4, it extends this result, providing a bound on the percentile of the ability distribution such that there exist stable governments that do not include any individuals with competences above this percentile.

Proposition 2 *Suppose $l \geq 1$ (and as before, that Assumptions 1, 2′, and 3 hold).*

1. *If*
$$n \geq 2k + k(k-l)\frac{(k-1)!}{(l-1)!\,(k-l)!}, \tag{11}$$
then there exists a stable government $G \in \mathcal{D}$ that contains no members of the ideal government G_1.

2. *Take any $x \in \mathbb{N}$. If*
$$n \geq k + x + x(k-l)\frac{(k-1)!}{(l-1)!\,(k-l)!}, \tag{12}$$

then for any set of individuals X with $|X| \leq x$, there exists a stable government $G \in \mathcal{D}$ such that $X \cap G = \varnothing$ (so no member of set X belongs to G).

3. *Suppose in addition that Assumption 4 holds and let*

$$\rho = \frac{1}{1 + (k - l)\frac{(k-1)!}{(l-1)!(k-l)!}}. \tag{13}$$

Then there exists a stable government $G \in \mathcal{D}$ that does not include any of the $\lfloor \rho n \rfloor$ highest ability individuals.

Proof. See Appendix A. ■

Let us provide the intuition for part 1 of Proposition 2 when $l = 1$. Recall that G_1 is the most competent government. Let G be the most competent government among those that do not include members of G_1 (such G exists, since $n > 2k$ by assumption). In this case, Proposition 2 implies that G is stable, that is, $G \in \mathcal{D}$. The reason is that if $\phi(G) = H \neq G$, then $\Gamma_H > \Gamma_G$, and therefore $H \cap G_1$ contains at least one element by construction of G. But then $\phi(H) = G_1$, as implied by (9). Intuitively, if $l = 1$, then once the current government contains a member of the most competent government G_1, this member will consent to (support) a transition to G_1, which will also receive the support of the population at large. She can do so, because G_1 is stable, thus there are no threats that further rounds of transitions will harm her. But then, as in Example 1 in the Introduction, G itself becomes stable, because any reform away from G will take us to an unstable government. Part 2 of the proposition has a similar intuition, but it states the stronger result that one can choose any subset of the society with size not exceeding the threshold defined in (12) such that there exist stable governments that do not include any member of this subset (which may be taken to include several of the most competent governments).[14] Finally, part 3, which follows immediately from part 2 under Assumption 4, further strengthens both parts 1 and 2 of this proposition and also parts 3 and 4 of Proposition 1; it shows that there exist stable governments that do not include a certain fraction of the highest ability individuals. Interestingly, this fraction, given in (13), is non-monotonic in l, reaching its maximum at $l = k/2$, i.e., for an "intermediate democracy". This partly anticipates the results pertaining to the relative successes of different regimes in selecting more competent governments, which we discuss in the next proposition.

We next turn to a more systematic discussion of this question, that is, of whether societies with a greater degree of democracy "perform better". For this analysis, we suppose that Assumption 4 holds. The next example, which is similar to Example 1 from the Introduction, shows

[14]Note that the upper bound on X in Part 2 of Proposition 2 is $O(x)$, meaning that increasing x does not require an exponential increase in the size of population n for Proposition 2 to hold.

that, starting with the same government, the long-run equilibrium government may be worse when political institutions are more democratic (as long as we are not in a perfect democracy).

Example 4 Take the setup from Example 3 ($n = 9$, $k = 3$, $l = 1$, and $m = 5$), and suppose that the initial government is $\{4,5,6\}$. As we showed there, government $\{4,5,6\}$ is stable, and will therefore persist. Suppose, however, that $l = 2$ instead. In that case, $\{4,5,6\}$ is unstable, and $\phi(\{4,5,6\}) = \{1,4,5\}$; thus there will be a transition to $\{1,4,5\}$. Since $\{1,4,5\}$ is more competent than $\{4,5,6\}$, this is an example where the long-run equilibrium government is worse under $l = 1$ (with "more democratic" institutions) than under $l = 2$ (with "less democratic" institutions). Note that if $l = 3$, $\{4,5,6\}$ would be stable again.

When either $k = 1$ or $k = 2$, the structure of stable governments is relatively straightforward. (Note that in this proposition, and in the examples that follow, a, b or c denote the indices of individuals, with our ranking that lower-ranked individuals have higher ability; thus $\gamma_a > \gamma_b$ whenever $a < b$.)

Proposition 3 *Suppose that Assumptions 1, 2′, 3 and 4 hold.*

1. *If $k = 1$, then either $l = 0$ (perfect democracy), in which case $\phi(G) = \{G_1\} = \{1\}$ for any $G \in \mathcal{G}$, or $l = 1 = k$ (extreme dictatorship), in which case $\phi(G) = G$ for any $G \in \mathcal{G}$.*

2. *If $k = 2$, and $l = 0$ (perfect democracy), then $\phi(G) = G_1 = \{1,2\}$ for any $G \in \mathcal{G}$. If $k = 2$ and $l = 1$ (imperfect democracy), then if $G = \{a,b\}$ with $a < b$, we have $\phi(G) = \{a-1,a\}$ when a is even and $\phi(G) = \{a, a+1\}$ when a is odd; in particular, $\phi(G) = G$ if and only if a is odd and $b = a+1$. If $k = 2$ and $l = 2$ (extreme dictatorship), then $\phi(G) = G$ for any $G \in \mathcal{G}$.*

Proof. See Appendix A. ■

Proposition 3, though simple, provides an important insight about the structure of stable governments that will be further exploited in the next section. In the case of an imperfect democracy, highlighted here with $k = 2$ and $l = 1$, the competence of the stable government is determined by the more able of the two members of the initial government. This means that, with rare exceptions, the quality of the initial government will improve to some degree, i.e., typically $\Gamma_{\phi(G)} > \Gamma_G$. However, this increase is generally limited; when $G = \{a,b\}$ with $a < b$, $\phi(G) = \{a-1,a\}$ or $\phi(G) = \{a,a+1\}$, so that at best the next highest ability individual is added to the initial government instead of the lower ability member. Therefore, summarizing these three cases, we can say that with a perfect democracy, the best government will arise; with an extreme dictatorship, there will be no improvement in the initial government; and with an imperfect democracy, there will be some limited improvements in the quality of the government.

When $k \geq 3$, the structure of stable governments is more complex, though we can still develop a number of results and insights about the structure of such governments. Naturally, the extremes with $l = 0$ (perfect democracy) and $l = 3$ (extreme dictatorship) are again straightforward. If $l = 1$ and the initial government is $G = \{a, b, c\}$, where $a < b < c$, then we can show that members ranked above $a - 2$ will never become members of the stable government $\phi(G)$, and the most competent member of G, a, is always a member of the stable government $\phi(G)$.[15] Therefore, again with $l = 1$, only incremental improvements in the quality of the initial government are possible. This ceases to be the case when $l = 2$. In this case, it can be shown that whenever $G = \{a, b, c\}$, where $a + b < c$, $\phi(G) \neq G$; instead $\phi(G) = \{a, b, d\}$, where $d < c$ and in fact, $d \ll a$ is possible. This implies a potentially very large improvement in the quality of the government (contrasting with the incremental improvements in the case where $l = 1$). Loosely speaking, the presence of two veto players when $l = 2$ allows the initial government to import very high ability individuals without compromising stability. The next example illustrates this feature, which is at the root of the result highlighted in Example 4, whereby greater democracy can lead to worse stable governments.

Example 5 Suppose $k = 3$, and first take the case where $l = 1$. Suppose $G = \{100, 101, 220\}$, meaning that the initial government consists of individuals ranked 100, 101, and 220 in terms of ability. Then $\phi_{l=1}(G) = \{100, 101, 102\}$, that is, there will be improvements in the quality of the third member of the government, but not in the quality of the highest ability member (and in this case, neither in the quality of the second highest ability member). More generally, recall that only very limited improvements in the quality of the highest ability member are possible in this case. Suppose instead that $l = 2$. Then it can be shown that $\phi_{l=2}(G) = \{1, 100, 101\}$, so that now the stable government includes the most able individual in the society. Naturally if the gaps in ability at the top of the distribution are larger, implying that highest ability individuals have a disproportionate effect on government competence, this feature becomes particularly valuable.

The following example extends the logic of Example 5 to any distribution and shows how, under certain configurations, expected competence is higher under $l = 2$ than $l = 1$ under any distribution over initial (feasible) governments.

Example 6 Suppose $k = 3$, and fix a (any) probability distribution over initial governments with full support (i.e., with a positive probability of picking any initial feasible government).

[15] More specifically, government $G = \{a, b, c\}$, where $a < b < c$, is stable if and only if $a = b - 1 = c - 2$, and c is a multiple of 3. Moreover, for any government $G = \{a, b, c\}$ with $a < b < c$, $\phi(G) = \{a - 2, a - 1, a\}$ if a is a multiple of 3, $\phi(G) = \{a - 1, a, a + 1\}$ if $a + 1$ is a multiple of 3, and $\phi(G) = \{a, a + 1, a + 2\}$ if $a + 2$ is a multiple of 3.

Assume that of players $i_1, \ldots, i_n$, the first q (where q is a multiple of 3 and $3 \leq q < n - 3$) are "smart," while the rest are "incompetent," so that governments that include at least one of players $i_1, \ldots, i_q$ will have very high competence relative to governments that do not. Moreover, differences in competence among governments that include at least one of the players $i_1, \ldots, i_q$ and also among those that do not are small relative to the gap between the two groups of governments. Then it can be shown that the expected competence of the stable government $\phi_{l=2}(G)$ (under $l = 2$) is greater than that of $\phi_{l=1}(G)$ (under $l = 1$)—both expectations are evaluated according to the probability distribution fixed at the outset. This is intuitive in view of the structure of stable governments under the two political regimes. In particular, if G includes at least one of $i_1, \ldots, i_q$, so do $\phi_{l=1}(G)$ and $\phi_{l=2}(G)$. But if G does not, then $\phi_{l=1}(G)$ will not include them either, whereas $\phi_{l=2}(G)$ will include one with positive probability, since the presence of two veto players will allow the incorporation of one of the "smart" players without destabilizing the government.

Conversely, suppose that Γ_G is very high if all its players are from $\{i_1, \ldots, i_q\}$, and very low otherwise. In that case, the expected competence of $\phi(G)$ will be higher under $l = 1$ than under $l = 2$. Indeed, if $l = 1$, the society will end up with a competent government if at least one of the players is from $\{i_1, \ldots, i_q\}$, while if $l = 2$, because there are now two veto players, there needs to be at least two "smart" players for a competent government to form (though, when $l = 2$, this is not sufficient to guarantee the emergence of a competent government either).

Examples 5 and 6 illustrate a number of important ideas. With a lower degree of democracy, in these examples with $l = 2$, there are more "veto" players, and this makes a greater number of governments near the initial government stable, leading to a higher probability of improvement in the competence of some of the members of the initial government. In contrast, with a higher degree of democracy, in these examples $l = 1$, fewer governments near the initial one are stable, thus incremental improvements are more likely. Consequently, when including a few high ability individuals in the government is very important, regimes with a lower degree of democracy perform better (naturally, this effect is non-monotonic, and the perfect democracy, $l = 0$, always performs best). Another important implication of these examples is that the situations in which regimes with lower degrees of democracy may perform better are not confined to some isolated instances. This feature applies for a broad class of configurations and for expected competences, evaluated by taking uniform or nonuniform distributions over initial feasible governments. Nevertheless, we will see that in stochastic environments, there will be a distinct advantage to political regimes with greater degrees of democracy, a phenomenon the intuition of which will also be illustrated using Examples 5 and 6.

5 Equilibria in Stochastic Environments

In this section, we introduce stochastic shocks to competences of different coalitions (or different individuals) in order to study the flexibility of different political institutions in their ability to adapt the nature and the composition of the government to changes in the underlying environment. Changes in the nature and structure of "high competence" governments may result from changes in the economic, political, or social environment, which may in turn require different types of government to deal with newly emerging problems. Our main results in this section establish strong links between the degree of democracy and the flexibility to adapt to changing environments.

Changes in the environment are modeled succinctly by allowing changes in the function $\Gamma_G^t : \mathcal{G} \to \mathbb{R}$, which determines the competence associated with each feasible government. Formally, we assume that at each t, with probability $1 - \delta$, there is no change in Γ_G^t from Γ_G^{t-1}, and with probability δ, there is a shock and Γ_G^t may change. In particular, following such a shock we assume that there exists a set of distribution functions $F_\Gamma\left(\Gamma_G^t \mid \Gamma_G^{t-1}\right)$ that gives the conditional distribution of Γ_G^t at time t as functions of Γ_G^{t-1}. The characterization of political equilibria in this stochastic environment is a very challenging task in general. However, we will show that when δ is sufficiently small, there is a characterization very similar to that in Theorem 1 and exploit this for illustrating the main substantive implications of stochastic shocks on the selection of governments.

In the rest of this section, we first generalize our definition of (Markov) political equilibrium to this stochastic environment and generalize Theorems 1 and 2 (for δ small). We then provide a systematic characterization of political transitions in this stochastic environment and illustrate the links between the degree of democracy and institutional flexibility.

5.1 Stochastic Political Equilibria

The structure of stochastic political equilibria is complicated in general because individuals need to consider the implications of current transitions on future transitions under a variety of scenarios. Nevertheless, when the likelihood of stochastic shocks, δ, is sufficiently small, as we have assumed here, then political equilibria must follow a logic similar to that given in Definition 1 in Section 3. Motivated by this reasoning, we introduce a simple definition of stochastic political equilibria (with low probabilities of changes in the environment). Appendix B shows that when the discount factor is high and stochastic shocks are sufficiently infrequent, MPE of the explicit-form game outlined there and our notions of (stochastic) political equilibrium are again equivalent.

To introduce the notion of *(stochastic Markov) political equilibrium*, let us first consider a set of mappings $\phi_{\{\Gamma_G\}} : \mathcal{G} \to \mathcal{G}$ defined as in (6), but now separately for each $\{\Gamma_G\}_{G\in\mathcal{G}}$. These mappings are indexed by $\{\Gamma_G\}$ to emphasize this dependence. Essentially, if the configuration of competences of different governments given by $\{\Gamma_G\}_{G\in\mathcal{G}}$ applied forever, we would be in a nonstochastic environment and $\phi_{\{\Gamma_G\}}$ would be the equilibrium transition rule, or simply the political equilibrium, as shown by Theorems 1 and 2. The idea underlying our definition for this stochastic environment with infrequent changes is that while the current configuration is $\{\Gamma_G\}_{G\in\mathcal{G}}$, $\phi_{\{\Gamma_G\}}$ will still determine equilibrium behavior, because the probability of a change in competences is sufficiently small (see Appendix B). When the current configuration is $\{\Gamma_G\}_{G\in\mathcal{G}}$, $\phi_{\{\Gamma_G\}}$ will determine political transitions, and if $\phi_{\{\Gamma_G\}}(G) = G$, then G will remain in power as a stable government. However, when a stochastic shock hits and $\{\Gamma_G\}_{G\in\mathcal{G}}$ changes to $\{\Gamma'_G\}_{G\in\mathcal{G}}$, then political transitions will be determined by the transition rule $\phi_{\{\Gamma'_G\}}$, and unless $\phi_{\{\Gamma'_G\}}(G) = G$, following this shock, there will be a transition to a new government, $G' = \phi_{\{\Gamma'_G\}}(G)$.

Definition 2 *Let the set of mappings $\phi_{\{\Gamma_G\}} : \mathcal{G} \to \mathcal{G}$ (a separate mapping for each configuration $\{\Gamma_G\}_{G\in\mathcal{G}}$) be defined by the following two conditions. When the configuration of competences is given by $\{\Gamma_G\}_{G\in\mathcal{G}}$, we have that for any $G \in \mathcal{G}$:*

(i) the set of players who prefer $\phi_{\{\Gamma_G\}}(G)$ to G (in terms of discounted utility) forms a winning coalition, i.e., $S = \left\{i \in \mathcal{I} : V_i\left(\phi_{\{\Gamma_G\}}(G) \mid \phi_{\{\Gamma_G\}}\right) > V_i\left(G \mid \phi_{\{\Gamma_G\}}\right)\right\} \in \mathcal{W}_G$;

(ii) there is no alternative government $H \in \mathcal{G}$ that is preferred both to a transition to $\phi_{\{\Gamma_G\}}(G)$ and to staying in G permanently, i.e., there is no H such that $S'_H = \left\{i \in \mathcal{I} : V_i\left(H \mid \phi_{\{\Gamma_G\}}\right) > V_i\left(\phi_{\{\Gamma_G\}}(G) \mid \phi_{\{\Gamma_G\}}\right)\right\} \in \mathcal{W}_G$ and $S''_H = \left\{i \in \mathcal{I} : V_i\left(H \mid \phi_{\{\Gamma_G\}}\right) > w_i(G)/(1-\beta)\right\} \in \mathcal{W}_G$ (alternatively, $|S'_H| < m_G$, or $|S'_H \cap G| < l_G$, or $|S''_H| < m_G$, or $|S''_H \cap G| < l_G$).

Then a set of mappings $\phi_{\{\Gamma_G\}} : \mathcal{G} \to \mathcal{G}$ constitutes a (stochastic Markov) political equilibrium *for an environment with sufficiently infrequent changes if there is a transition to government G_{t+1} at time t (starting with government G_t) if and only if $\{\Gamma^t_G\}_{G\in\mathcal{G}} = \{\Gamma_G\}_{G\in\mathcal{G}}$ and $G_{t+1} = \phi_{\{\Gamma_G\}}(G_t)$.*

Therefore, a political equilibrium with sufficiently infrequent changes involves the same political transitions (or the stability of governments) as those implied by the mappings $\phi_{\{\Gamma_G\}}$ defined in (6), applied separately for each configuration $\{\Gamma_G\}$.

The next theorem provides the general characterization of stochastic political equilibria in environments with sufficiently infrequent changes.

Theorem 3 *Suppose that Assumptions 1-3 hold, and let $\phi_{\{\Gamma_G\}} : \mathcal{G} \to \mathcal{G}$ be the mapping defined by (6) applied separately for each configuration $\{\Gamma_G\}$. Then there exists $\beta_0 < 1$ such that for any discount factor $\beta > \beta_0$, $\phi_{\{\Gamma_G\}}$ is the unique acyclic political equilibrium.*

Proof. See Appendix A. ■

The intuition for this theorem is straightforward. When shocks are sufficiently infrequent, the same calculus that applied in the nonstochastic environment still determines preferences because all agents put most weight on the events that will happen before such a change. Consequently, a stable government will arise and will remain in place until a stochastic shock arrives and changes the configuration of competences. Following such a shock, the stable government for this new configuration of competences emerges. Therefore, Theorem 3 provides us with a tractable way of characterizing stochastic transitions. In the next subsection, we use this result to study the links between different political regimes and institutional flexibility.

5.2 The Structure of Stochastic Transitions

In the rest of this section, we compare different political regimes in terms of their flexibility (adaptability to stochastic shocks). Our main results will be that, even though a greater degree of democracy does not guarantee the emergence of more competent governments in the nonstochastic environment (nor does it guarantee greater expected competence), in the presence of shocks institutional structures with a greater degree of democracy (less incumbency advantage) will be more "flexible" ("will perform better in the long run"). In the results that follow, we always impose Assumptions 1, 2′, 3 and 4 (which again ensure uniqueness of acyclic political equilibria).

We also impose some additional structure on the distribution $F_\Gamma\left(\Gamma_G^t \mid \Gamma_G^{t-1}\right)$ by assuming that any shock corresponds to a rearrangement ("permutation") of the abilities of different individuals. Put differently, we assume throughout this subsection that there is a fixed vector of abilities, say $a = \{a_1, ..., a_n\}$, and the actual distribution of abilities across individuals at time t, $\left\{\gamma_j^t\right\}_{j=1}^n$, is given by some permutation φ^t of this vector a. We adopt the convention that $a_1 > a_2 > ... > a_n$. Intuitively, this captures the notion that a shock will change which individual is best placed to solve certain tasks and thus most effective in government functions.

The next proposition shows the difference in flexibility implied by different political regimes. Throughout the rest of this section, our measure of "flexibility" is the probability with which the best government will be in power (either at given t or as $t \to \infty$).[16] More formally, let $\pi_t\left(l, k, n \mid G, \{\Gamma_G\}\right)$ be the probability that in a society with n individuals under a political regime characterized by l (for given k), a configuration of competence given by $\{\Gamma_G\}$, and current government $G \in \mathcal{G}$, the most competent government will be in power at the time t. Given n and k, we will think of a regime characterized by l' as more flexible than one characterized by l if

[16] This is a natural metric of flexibility in the context of our model; since we have not introduced any cardinal comparisons between the abilities of individuals, "expect competence" would not be a meaningful measure (see also footnote 18). Note also that we would obtain similar results if we related flexibility to the probability that one of the best two or three governments comes to power, etc.

$\pi_t(l', k, n \mid G, \{\Gamma_G\}) > \pi_t(l, k, n \mid G, \{\Gamma_G\})$ for all G and $\{\Gamma_G\}$ and for all t following a stochastic shock. Similarly, we can think of the regime as asymptotically more flexible than another, if $\lim_{t\to\infty} \pi_t(l', k, n \mid G, \{\Gamma_G\}) > \lim_{t\to\infty} \pi_t(l, k, n \mid G, \{\Gamma_G\})$ for all G and $\{\Gamma_G\}$ (provided that these limits are well defined). Clearly, "being more flexible" is a partial order.

Proposition 4 1. *If $l = 0$ (i.e., perfect democracy), then a shock immediately leads to the replacement of the current government by the new most competent government.*

2. *If $l = 1$ (i.e., imperfect democracy), the competence of the government following a shock never decreases further; instead, it increases with probability no less than*

$$1 - \frac{(k-1)!\,(n-k)!}{(n-1)!} = 1 - \binom{n-1}{k-1}^{-1}.$$

Starting with any G and $\{\Gamma_G\}$, the probability that the most competent government will ultimately come to power as a result of a shock is

$$\lim_{t\to\infty} \pi_t(l, k, n \mid G, \{\Gamma_G\}) = \pi(l, k, n) \equiv 1 - \binom{n-k}{k}\binom{n}{k}^{-1} < 1.$$

For fixed k as $n \to \infty$, $\pi(l, k, n) \to 0$.

3. *If $l = k \geq 2$ (i.e., extreme dictatorship), then a shock never leads to a change in government. The probability that the most competent government is in power at any given period (any t) after the shock is*

$$\pi_t(l = k, k, n \mid \cdot, \cdot) = \binom{n}{k}^{-1}.$$

This probability is strictly less than $\pi_t(l = 0, k, n \mid G, \{\Gamma_G\})$ and $\pi_t(l = 1, k, n \mid G, \{\Gamma_G\})$ for any G and $\{\Gamma_G\}$.

Proof. See Appendix A. ∎

Proposition 4 contains a number of important results. A perfect democracy ($l = 0$) does not create any barriers against the installation of the best government at any point in time. Hence, under a perfect democracy every shock is "flexibly" met by a change in government according to the wishes of the population at large (which here means that the most competent government will come to power). As we know from the analysis in Section 4, this is no longer true as soon as members of the governments have incumbency advantage. In particular, we know that without stochastic shocks, arbitrarily incompetent governments may come to power and remain in power. However, in the presence of shocks the evolution of equilibrium governments becomes more complex.

Next consider the case with $l \geq 1$. Now, even though the immediate effect of a shock may be a deterioration in government competence, there are forces that increase government competence in the long run. This is most clearly illustrated in the case where $l = 1$. With this set of political institutions, there is zero probability that there will be a further decrease in government competence following a shock. Moreover, there is a positive probability that competence will improve and in fact a positive probability that, following a shock, the most competent government will be instituted. In particular, a shock may make the current government unstable, and in this case, there will be a transition to a new stable government. A transition to a less competent government would never receive support from the population. The change in competences may be such that the only stable government after the shock, starting with the current government, may be the best government.[17] Proposition 4 also shows that when political institutions take the form of an extreme dictatorship, there will never be any transition, thus the current government can deteriorate following shocks (in fact, it can do so significantly).

Most importantly, part 3 of Proposition 4 shows that imperfect democracy has a higher degree of *flexibility* than dictatorship, ensuring better long-run outcomes (and naturally perfect democracy has the highest degree of flexibility). This unambiguous ranking between imperfect democracy and dictatorship in the presence of stochastic shocks (and its stronger version stated in the next proposition) contrasts with the results in Section 4, which showed that general comparisons between imperfect democracy and dictatorship are not possible in the nonstochastic case. Thus, a distinct advantage of more democratic regimes might be their flexibility in the face of changing environments.

An informal intuition for the greater flexibility of more democratic regimes in the presence of stochastic shocks can be obtained from Examples 5 and 6 in the previous section. Recall from these examples that an advantage of the less democratic regime, $l = 2$, is that the presence of two veto players makes a large number of governments near the initial one stable. But this implies that if the initial government is destabilized because of a shock, there will only be a move to a nearby government. In contrast, the more democratic regime, $l = 1$, often makes highly incompetent governments stable because there are no nearby stable governments (recall, for example, part 2 of Proposition 3). But this also implies that if a shock destabilizes the current government, a significant improvement in the quality of the government becomes more likely. Thus, at a broad level, regimes with lower degrees of democracy "create more stability," facilitating small or moderate-sized improvements in initial government quality, but do not create a large "basin of attraction" for the highest competence governments (in particular, the best government). In contrast, in regimes with higher degrees of democracy, low competence governments are often made stable by the instability of nearby alternative governments; this instability can be a disadvantage in deterministic environments as illustrated in the previous

[17] Nevertheless, the probability of the most competent government coming to power, though positive, may be arbitrarily low.

section, but turns into a significant flexibility advantage in the presence of stochastic shocks because it creates the possibility that, after a shock, there may be a jump to a very high competence government (in particular, to the best government, which now has a larger "basin of attraction" than in less democratic regimes).

The next proposition strengthens the conclusions of Proposition 4. In particular, it establishes that the probability of having the most competent government in power is increasing in the degree of democracy more generally (i.e., it is decreasing in l).[18]

Proposition 5 *The probability of having the most competent government in power after a shock (for any t), $\pi_t(l, k, n \mid G, \{\Gamma_G\})$, is decreasing in l for any k, n, G and $\{\Gamma_G\}$. That is, more democratic regimes are more likely to produce the most competent government.*

Proof. See Appendix A. ■

The results of Propositions 4 and 5 can be strengthened further, when shocks are "limited" in the sense that only the abilities of two (or in the second part of the proposition, of $x \geq 2$) individuals in the society are swapped. The next proposition contains these results.

Proposition 6 *Suppose that any shock permutes the abilities of x individuals in the society.*

1. *If $x = 2$ (so that the abilities of two individuals are swapped at a time) and $l \leq k - 1$, then the competence of the government in power is nondecreasing over time, that is, $\pi_t(l, k, n \mid G, \{\Gamma_G\})$ is nondecreasing in t for any l, k, n, G and $\{\Gamma_G\}$ such that $l \leq k-1$. Moreover, if the probability of swapping of abilities between any two individuals is positive, then the most competent government will be in power as $t \to \infty$ with probability 1, that is, $\lim_{t\to\infty} \pi_t(l, k, n \mid G, \{\Gamma_G\}) = 1$ (for any l, k, n, G and $\{\Gamma_G\}$ such that $l \leq k-1$).*
2. *If $x > 2$, then the results in part 1 hold provided that $l \leq k - \lfloor x/2 \rfloor$.*

Proof. See Appendix A. ■

An interesting application of Proposition 6 is that when shocks are (relatively) rare and limited in their scope, relatively democratic regimes will gradually improve over time and install the most competent government in the long run. This is not true for the most autocratic governments, however. This proposition, therefore, strengthens the conclusions of Propositions 4 and 5 in highlighting the flexibility benefits of more democratic regimes.

[18] This conclusion need not be true for "expected competence" of the government, since we have not made "cardinal" assumptions on abilities. In particular, it is possible that some player is not a member of any stable government for some l and becomes part of a stable government for some $l' < l$. If this player has very low ability, then expected competence under l' may be lower. In Appendix B, we provide an example (Example 10) illustrating this point, and we also show that expected competence of government is monotone in l when l is close to 0 or to k.

The political institutions considered so far are "junta-like" in the sense that no specific member is indispensable. Incumbency advantage takes the form of the requirement that some members of the current government must consent to change. The alternative is a "royalty-like" environment where one or several members of the government are irreplaceable. All else equal, this can be conjectured to be a negative force, since it would mean that a potentially low ability person must always be part of the government. However, because such an irreplaceable member (the member of the "royalty") is also unafraid of changes, better governments may be more likely to arise under certain circumstances, whereas, as we have seen, junta members would resist certain changes because of the further transitions that these will unleash.

More formally, we change Assumption 2 and the structure of the set of winning coalitions $\mathcal{W}_G$ to accommodate "royalty-like" situations. We assume that there are l royalty individuals whose votes are *always* necessary for a transition to be implemented (regardless of whether they are current government members). We denote the set of these individuals by Y. So, the new set of winning coalitions becomes

$$\mathcal{W}_G = \{X \in \mathcal{C} : |X| \geq m \text{ and } Y \subset X\}.$$

We also assume that all royal individuals are members of the initial government, that is, $Y \subset G^0$. The next proposition characterizes the structure of equilibrium in this case and compares royalty-like and junta-like institutions in terms of the expected competence of the equilibrium (stable) government.

Proposition 7 *Suppose that we have a royalty-system with $1 \leq l < k$ and competences of governments are given by* (10)*. so that the l royals are never removed from the government. If $\{a_1, ..., a_n\}$ is sufficiently "convex" meaning that $\frac{a_1 - a_2}{a_2 - a_n}$ is sufficiently large, then the expected competence of the government under the royalty system is greater than under the original, junta-like system (with the same l). The opposite conclusion holds if $\frac{a_1 - a_{n-1}}{a_{n-1} - a_n}$ is sufficiently low and $l = 1$.*

Proof. See Appendix A. ∎

Proposition 7 shows that royalty-like regimes perform better in the face of shocks than junta-like regimes in terms of generating higher expected competence governments, provided that $\{a_1, ..., a_n\}$ is highly "convex" (such convexity implies that the benefit to society from having the highest ability individual in government is relatively high). As discussed above, juntas are unlikely to lead to such high quality governments because of the fear of a change leading to a further round of changes, excluding all initial members of the junta. Royalty-like regimes avoid this fear. Nevertheless, royalty-like regimes have a disadvantage in that the ability of royals may be very low or may change at some point and become very low, and the royals will always be part of the government. In this sense, royalty-like regimes create a clear disadvantage. However, this result shows that when $\{a_1, ..., a_n\}$ is sufficiently convex (so as to outweigh the loss of expected competence because of the presence of a potentially low ability royal), expected competence is nonetheless higher under the royalty-like system. This result is interesting because it suggests that different types of dictatorships may have distinct implications for long-run quality of government and performance, and regimes that provide security to certain members of the incumbent government may be better at dealing with changes and in ensuring relatively high-quality governments in the long run.

6 Conclusion

In this paper, we provided a tractable dynamic model of political selection. The main barrier to the selection of good politicians and to the formation of good governments in our model is not the difficulty of identifying competent or honest politicians, but the incumbency advantage of current governments. Our framework shows how a small degree of incumbency advantage can lead to the persistence of highly inefficient and incompetent governments. This is because incumbency advantage implies that one of (potentially many) members of the government needs to consent to a change in the composition of government. However, all current members of the government may recognize that any change may unleash a further round of changes, ultimately unseating themselves. In this case, they will all oppose any change in government, even if such changes can improve welfare significantly for the rest of the society, and highly incompetent governments can remain in power.

Using this framework, we study the implications of different political institutions for the selection of governments both in nonstochastic and stochastic environments. A perfect democracy corresponds to a situation in which there is no incumbency advantage, thus citizens can nominate alternative governments and vote them to power without the need for the consent of any member of the incumbent government. In this case, we show that the most competent government will always come to power. However, interestingly, any deviation from perfect democracy breaks this result and the long-run equilibrium government can be arbitrarily incompetent (relative to the best possible government). In extreme dictatorship, where any single member of the current government has a veto power on any change, the initial government always remains in power and this can be arbitrarily costly for the society. More surprisingly, the same is true for any political institution other than perfect democracy. Moreover, there is no obvious ranking between different sets of political institutions (other than perfect democracy and extreme dictatorship) in terms of what they will imply for the quality of long run government. In fact, regimes with lower degrees of democracy (greater incumbency advantage) can lead to higher quality governments both in specific instances and in expectation (with uniform or non-uniform distribution over the set of feasible initial governments). Even though no such ranking across political institutions is possible, we provide a fairly tight characterization of the structure of stable governments in our benchmark nonstochastic society.

In contrast, in stochastic environments, more democratic political regimes have a distinct advantage because of their greater "flexibility". In particular, in stochastic environments, either the abilities and competences of individuals or the needs of government functions change, shuffling the ranking of different possible governments in terms of their competences and effectiveness. A greater degree of democracy then ensures greater "adaptability" or flexibility. Perfect democracy is most flexible and immediately adjusts to any shock by installing a new government that has the greatest competence after the shock. Extreme dictatorship is the polar opposite and again leads to no change in the initial government. Therefore, shocks that reduce the competence of the individuals currently in power can lead to significant deterioration in the quality of government. Most interestingly, more democratic political institutions allow a greater degree of institutional flexibility in response to shocks. In particular, we show that shocks cannot lead to the emergence of a worse government (relative to the competence of the government in power following the shock). They may, however, destabilize the current government and induce the emergence of a more competent government. We show that political institutions with a greater degree of democracy have higher probability of improving the competence of the government following a shock and ultimately installing the most competent government.

Finally, we also compare "junta-like" and "royalty-like" regimes. The former is our benchmark society, where change in government requires the consent or support of one or multiple members of the current government. The second corresponds to situations in which one or multiple individuals are special and must always be part of the government (hence the title "royalty"). If royal individuals have low ability, royalty-like regimes can lead to the persistence of highly incompetent governments. However, we also show that in stochastic environments, royalty-like regimes may lead to the emergence of higher quality governments in the long run than junta-like regimes. This is because royal individuals are not afraid of changes in governments, because their powers are absolute. In contrast, members of the junta may resist changes that may increase government competence or quality because such changes may lead to another round of changes, ultimately excluding all members of the initial government.

An important contribution of our paper is to provide a tractable framework for the dynamic analysis of the selection of politicians. This tractability makes it possible to extend the analysis in various fruitful directions. For example, it is possible to introduce conflict of interest among voters, for example, by having each government be represented by two characteristics: competence and ideological leaning. In this case, not all voters will simply prefer the most competent government. The general approach developed here remains applicable in this case. Another generalization would allow the strength of the preferences of voters towards a particular government to influence the number of veto players, so that a transition away from a semi-incompetent government can be blocked by a few insiders, but more unified opposition from government members would be necessary to block a transition away from a highly incompetent government.

An open question, which would be interesting to investigate perhaps by placing more structure on preferences and institutions, is the characterization of equilibria in the stochastic environment when shocks occur frequently. This case introduces a number of nontrivial technical difficulties, and even existence of pure strategy equilibria becomes difficult to guarantee.

The most important direction for future research, which is again feasible using the general approach here, is an extension of this framework to incorporate the asymmetric information issues emphasized in the previous literature. For example, we can generalize the environment in this paper such that the ability of an individual is not observed until she becomes part of the government. In this case, to install high quality governments, it is necessary to first "experiment" with different types of governments. The dynamic interactions highlighted by our analysis will then become a barrier to such experimentation. In this case, the set of political institutions that will ensure high quality governments must exhibit a different type of flexibility, whereby some degree of "churning" of governments can be guaranteed even without shocks. Another interesting area is to introduce additional instruments, so that some political regimes can provide incentives to politicians to take actions in line with the interests of the society at large. In that case, successful political institutions must ensure both the selection of high ability individuals and the provision of incentives to these individuals once they are in government. Finally, as hinted by the discussion in this paragraph, an interesting and challenging extension is to develop a more general "mechanism design" approach in this context, whereby certain aspects of political institutions are designed to facilitate the appropriate future changes in government. We view these directions as interesting and important areas for future research.

Appendix A

In the appendix, we use the following notation. First, we introduce the following binary relation on the set of feasible governments $\mathcal{G}$. For any $G, H \in \mathcal{G}$ we write

$$H \succ G \text{ if and only if } \{i \in \mathcal{I} : w_i(H) > w_i(G)\} \in \mathcal{W}_G. \tag{A1}$$

In other words, $H \succ G$ if and only if there exists a winning coalition in G such that all members of G have higher stage payoff in H than in G. Let us also define set $\mathcal{D}$ as

$$\mathcal{D} = \{G \in \mathcal{G} : \phi(G) = G\}.$$

The next two lemmas summarize the properties of payoff functions and mapping ϕ.

Lemma 1 *Suppose that $G, H \in \mathcal{G}$ and $\Gamma_G > \Gamma_H$. Then:*

1. *If for $i \in \mathcal{I}$, $w_i(G) < w_i(H)$, then $i \in H \setminus G$.*
2. *$H \nsucc G$.*
3. *$|\{i \in \mathcal{I} : w_i(G) > w_i(H)\}| > n/2 \geq \bar{k}$.*

Proof of Lemma 1. Part 1. If $\Gamma_G > \Gamma_H$ then, by Assumption 1, $w_i(G) > w_i(H)$ whenever $i \in G$ or $i \notin H$. Hence, $w_i(G) < w_i(H)$ is possible only if $i \in H \setminus G$ (note that $w_i(G) = w_i(H)$ is ruled out by Assumption 3). At the same time, $i \in H \setminus G$ implies $w_i(G) < w_i(H)$ by Assumption 1, hence the equivalence.

Part 2. We have $|H \setminus G| \leq |H| \leq \bar{k} \leq m_G$; since by Assumption 2 $\bar{k} \leq m_G$, then $H \setminus G \notin \mathcal{W}_G$, and $H \nsucc G$ by definition (A1).

Part 3. We have $\{i \in \mathcal{I} : w_i(G) > w_i(H)\} = \mathcal{I} \setminus \{i \in \mathcal{I} : w_i(G) < w_i(H)\} \supset \mathcal{I} \setminus (H \setminus G)$, hence, $|\{i \in \mathcal{I} : w_i(G) > w_i(H)\}| \geq n - \bar{k} \geq n - n/2 = n/2 \geq \bar{k}$. ∎

Lemma 2 *Consider the mapping ϕ defined in (6) and let $G, H \in \mathcal{G}$. Then:*

1. *Either $\phi(G) = G$ (and then $G \in \mathcal{D}$) or $\phi(G) \succ G$.*
2. *$\Gamma_{\phi(G)} \geq \Gamma_G$.*
3. *If $\phi(G) \succ G$ and $H \succ G$, then $\Gamma_{\phi(G)} \geq \Gamma_H$.*
4. *$\phi(\phi(G)) = \phi(G)$.*

Proof of Lemma 1. The proof of this lemma is straightforward and is omitted. ■

Proof of Theorem 1. As in the text, let us enumerate elements of $\mathcal{G}$ as $\{G_1, G_2, ..., G_{|\mathcal{G}|}\}$ such that $\Gamma_{G_x} > \Gamma_{G_y}$ whenever $x < y$. First, we prove that the function ϕ defined in (6) constitutes a (Markov) political equilibrium. Take any G_q, $1 \le q \le |\mathcal{G}|$. By (6), either $\phi(G_q) = G_q$ or $\phi(G_q) \in \mathcal{M}_q$. In the latter case, the set of players who obtain higher stage payoff under $\phi(G_q)$ than under G_q (i.e., those with $w_i(\phi(G_q) > w_i(G_q)))$ form a winning coalition in G_q by (5). Since by definition $\phi(G_q)$ is ϕ-stable, we have $V_i(\phi(G_q)) > V_i(G_q)$ for a winning coalition of players. Hence, in either case condition (i) of Definition 1 is satisfied.

Now, suppose, to obtain a contradiction, that condition (ii) of Definition 1 is violated, and $X, Y \in \mathcal{W}_{G_q}$ are winning coalitions such that $V_i(H) > w_i(G_q)/(1-\beta)$ for all $i \in X$ and $V_i(H) > V_i(\phi(G_q))$ for all $i \in Y$ and some alternative $H \in \mathcal{G}$. Consider first the case $\Gamma_{\phi(H)} \neq \Gamma_{\phi(G_q)}$, then $V_i(H) > V_i(\phi(G_q))$ would imply $w_i(\phi(H)) > w_i(\phi(G_q))$ as β is close to 1, and hence the set of players who have $w_i(\phi(H)) > w_i(\phi(G_q))$ would include Y, and thus would be a winning coalition in G_q. This is impossible if $\Gamma_{\phi(H)} < \Gamma_{\phi(G_q)}$ (only players in $\phi(H)$ would possibly prefer $\phi(H)$, and they are less than m_{G_q}). If $\Gamma_{\phi(H)} > \Gamma_{\phi(G_q)}$, however, we would get a government $\phi(H)$ which is ϕ-stable by construction of ϕ and which is preferred to G_q by at least m_{G_q} players (all except perhaps members of G_q) and at least l_G members of G_q (indeed, at least l_G members of G_q – those in coalition X – had $V_i(\phi(H)) > w_i(G_q)/(1-\beta)$, which then means they belong to $\phi(H)$, and hence must have $w_i(\phi(H)) > w_i(G_q))$ as $\phi(H)$ is stable and $\Gamma_{\phi(G_q)} \ge \Gamma_{G_q}$. This would imply that $\phi(H) \in \mathcal{M}_q$ by (5), but in that case $\Gamma_{\phi(H)} > \Gamma_{\phi(G_q)}$ would contradict (6).

Finally, consider the case $\Gamma_{\phi(H)} = \Gamma_{\phi(G_q)}$, which by Assumption 3 implies $\phi(H) = \phi(G_q)$. Now $V_i(H) > V_i(\phi(G_q))$ implies $w_i(H) > w_i(\phi(G_q))$ for all $i \in Y$, as the instantaneous utilities are the same except for the current period. Since this includes at least m_{G_q} players, we must have that $\Gamma_H > \Gamma_{\phi(G_q)}$. But $\Gamma_{\phi(H)} \ge \Gamma_H$ by (6), so $\Gamma_{\phi(H)} \ge \Gamma_H > \Gamma_{\phi(G_q)}$, which contradicts $\Gamma_{\phi(H)} = \Gamma_{\phi(G_q)}$. This contradiction proves that mapping ϕ satisfies both conditions of Definition 1, and thus forms a political equilibrium.

To prove uniqueness, suppose that there is another acyclic political equilibrium ψ. For each $G \in \mathcal{G}$, define $\chi(G) = \psi^{|\mathcal{G}|}(G)$; due to acyclicity, $\chi(G)$ is ψ-stable for all G. We prove the following sequence of claims. First, we must have that $\Gamma_{\chi(G)} \ge \Gamma_G$; indeed, otherwise condition (i) of Definition 1 would not be satisfied for large β.

Second, we prove that all transitions must take place in one step, i.e., $\chi(G) = \psi(G)$ for all G. If this were not the case, then, due to finiteness of any chain of transitions there would exist $G \in \mathcal{G}$ such that $\psi(G) \neq \chi(G)$ but $\psi^2(G) = \chi(G)$. Take $H = \chi(G)$. Then $\Gamma_H > \Gamma_{\psi(G)}$, $\Gamma_H > \Gamma_G$ and $\psi(H) = H$. For β sufficiently close to 1, the condition $V_i(H) > w_i(G)/(1-\beta)$ is automatically satisfied for the winning coalition of players who had $V_i(\phi(G)) > w_i(G) / (1-\beta)$. We next prove that $V_i(H) > V_i(\phi(G))$ for a winning coalition of players in G. Note that this condition is equivalent to $w_i(H) > w_i(\phi(G))$. The fact that at least m_G players prefer H to $\phi(G)$ follows from $\Gamma_H > \Gamma_{\psi(G)}$. Moreover, since $\chi(G) = H$, at least l_G members of G must also be members of H; naturally, they prefer H to $\phi(G)$. Consequently, condition (ii) of Definition 1 is violated. This contradiction proves that $\chi(G) = \psi(G)$ for all G.

Finally, we prove that $\psi(G)$ coincides with $\phi(G)$ defined in (6). Suppose not, i.e., that $\phi(G) \neq \psi(G)$. Without loss of generality, we may assume that G is the most competent government that has $\phi(G) \neq \psi(G)$, i.e., $\phi(H) = \psi(H)$ whenever $\Gamma_H > \Gamma_G$. By Assumption 3, we have that $\Gamma_{\phi(G)} \neq \Gamma_{\psi(G)}$. Suppose that $\Gamma_{\phi(G)} > \Gamma_{\psi(G)}$ (the case $\Gamma_{\phi(G)} < \Gamma_{\psi(G)}$ is treated similarly). As $\psi(G)$ forms a political equilibrium, it must satisfy condition (ii) of Definition 1, for $H = \phi(G)$ in particular. Since $\phi(G)$ is a political equilibrium, it must be that $w_i(\phi(G)) > w_i(G)$, and thus $V_i(H \mid \phi) > w_i(G) / (1-\beta)$, for a winning coalition of players. Now, we see that the following two facts must hold. First, $V_i(H \mid \psi) > w_i(G) / (1-\beta)$ for a winning coalition of players; this follows from the fact that H is ϕ-stable and thus ψ-stable as $\Gamma_H > \Gamma_G$ and from the choice of G. Second, $V_i(H \mid \psi) > V_i(\psi(G) \mid \psi)$ for a winning coalition of players; indeed, H and $\psi(G)$ are ψ-stable, and the former is preferred to the latter (in terms of states payoffs) by at least m_G players and at least l_G government members, as $\Gamma_{\phi(G)} > \Gamma_{\psi(G)}$ and the intersection of H and G contains at least l_G members (since $H = \phi(G)$). The existence of such H leads to a contradiction which completes the proof. ■

Proof of Theorem 2. Part 1. We prove the statement for the case $l = 0$. The case $l \geq 1$ is covered by Part 2 of the theorem.

To obtain a contradiction, suppose that there is a cycle; this implies that so that there are $q \geq 2$ different governments $H_1, \ldots, H_q$ such that $\phi(H_j) = H_{j+1}$ for all $1 \leq j < q$, and $\phi(H_q) = H_1$. Without loss of generality, let H_1 be the least competent of these governments. Take $H = \phi(H_2)$ (if $q > 2$ then $H = H_3$ and if $q = 2$ then $H = H_1$). As ϕ is a political equilibrium, $V_i(H) > V_i(H_2)$ holds for a winning coalition in H_2. But winning coalitions are the same for all governments, so $V_i(H) > V_i(H_2)$ holds for a winning coalition in H_1. Moreover, by Assumption 1, $V_i(H) > w_i(H_1) / (1-\beta)$ for all players except, perhaps, members of H_1 (as H_1 is the least competent government). However, the existence of such alternative H contradicts that ϕ is a political equilibrium, as the condition (ii) of Definition 1 is violated for government H_1. This contradiction completes the proof.

Part 2. Suppose to obtain a contradiction that there is a cycle, so that there are $q \geq 2$ different governments $H_1, \ldots, H_q$ such that $\phi(H_j) = H_{j+1}$ for all $1 \leq j < q$, and $\phi(H_q) = H_1$. Without loss of generality, let H_1 be the most competent of these governments. Take any

$i \in H_1$. In that case, $V_i(H_1) > V_i(H_2)$, as player i gets the highest utility under H_1 (formally, we have $V_i(H_2) < w_i(H_1)/(1-\beta)$ as H_1 is i's most preferred government in the cycle, hence $w_i(H_1) + \beta V_i(H_2) > V_i(H_2)$, which means $V_i(H_1) > V_i(H_2)$). Since this holds for all members of H_1 and $l_{H_1} \geq 1$, it is impossible that for a winning coalition of players in H_1 the condition $V_i(H_2) > V_i(H_1)$ is satisfied. This contradiction completes the proof.

Part 3. Suppose to obtain a contradiction that the statement does not hold. As the number of mappings $\mathcal{G} \to \mathcal{G}$ is finite, there exists mapping ϕ which forms a cyclic political equilibrium for β arbitrarily close to 1. Moreover, since the number of coalitions is finite, we can only consider β in which ϕ is supported by the same coalitions in players. Let $H_1, \ldots, H_q$ with $q \geq 2$ satisfy $\phi(H_j) = H_{j+1}$ for all $1 \leq j < q$ and $\phi(H_q) = H_1$, so that the sequence constitutes a cycle for ϕ. This means, in particular, that for any $G \in \{H_1, \ldots, H_q\}$, the inequality $V_i(\phi(G) \mid \phi, \beta) > V_i(G \mid \phi, \beta)$ is satisfied for the same winning coalition of players for some β arbitrarily close to 1. As this inequality is equivalent to $w_i(G) < (1-\beta) V_i(\phi(G) \mid \phi, \beta)$, we must have (by taking the limit as $\beta \to 1$) that

$$w_i(G) \leq u_i \equiv \frac{1}{q} \sum_{p=1}^{q} w_i(H_p),$$

where the last the quality defines u_i.

Without loss of generality, suppose that H_2 is the least competent of the governments in the cycle. Consider first the case $q = 2$. We must have that

$$w_i(H_1) \leq \frac{w_i(H_1) + w_i(H_2)}{2},$$

and hence $w_i(H_1) \leq w_i(H_2)$, for a winning coalition of players in H_1. However, only members of H_2 may satisfy these inequalities, and their number is less than m_{H_1}. We get an immediate contradiction.

Consider now the more complicated case $q \geq 3$. In this case, H_1, H_2, H_3 are three different governments. We take $H = H_3$ and show that the condition (ii) of Definition 1 is violated for current government H_1 and alternative $H = H_3$. In particular, we need to check that the following two conditions are satisfied:

1. $V_i(H_3) > V_i(H_2)$ for a winning coalition of players. Note that this is equivalent to $w_i(H_2) < V_i(H_3)$. The latter is satisfied for sufficiently large β provided that $w_i(H_2) < u_i$. This last inequality holds for all players except, perhaps, members of H_2, i.e., for at least m_{H_1} of them. Therefore, we just have to prove that $w_i(H_2) < u_i$ for at least l_{H_1} members of H_1. However, we know that $w_i(H_1) \leq u_i$ for at least l_{H_1} members of H_1. Moreover, since they belong to H_1 and $\Gamma_{H_2} < \Gamma_{H_1}$, we must have that $w_i(H_2) < w_i(H_1)$ by Assumption 1 for each of these players. But this immediately implies $w_i(H_2) < u_i$.

2. $V_i(H_3) > w_i(H_1)/(1-\beta)$ for a winning coalition of players. Suppose not; then there must exist player i such that $(1-\beta)V_i(H_3) \leq w_i(H_1) < (1-\beta)V_i(H_2)$ (since the latter inequality holds for a winning coalition of players). Taking the limit, we get $u_i \leq w_i(H_1) \leq u_i$, and hence $w_i(H_1) = u_i$. However, this contradicts the assumption.

Consequently, we found $H = H_3$ for which the condition (ii) of Definition 1 is violated. This contradiction completes the proof.

Part 4. The proof of this part follows the proof of Part 3, except the steps involved in checking the condition $V_i(H_3) > w_i(H_1)/(1-\beta)$ for a winning coalition of players (indeed, it is only in this step that the assumption in Part 3 was used). To check that last condition, again suppose that it did not hold. Then there must exist player i such that $(1-\beta)V_i(H_3) \leq w_i(H_1) < (1-\beta)V_i(H_2)$ (since the latter inequality holds for a winning coalition of players). Taking the limit, we get $u_i \leq w_i(H_1) \leq u_i$, and hence $w_i(H_1) = u_i$. Given the assumption, this is only possible if player i is either a member of all governments $H_1, \ldots, H_q$ or a member of none of them (otherwise $w_i(H_1) > u_i$ if $i \in H_1$ and $w_i(H_1) < u_i$ if $i \notin H_1$). In both cases, $w_i(H_2) < w_i(H_1)$, and thus $w_i(H_2) < u_i$. But then $V_i(H_3) > V_i(H_2)$ if β is sufficiently close to 1. This contradicts the double inequality above, which proves that $V_i(H_3) > w_i(H_1)/(1-\beta)$ for a winning coalition.

Consequently, we found $H = H_3$ for which the condition (ii) of Definition 1 is violated. This contradiction completes the proof. ■

Proof of Proposition 1. Part 1. Suppose, to obtain a contradiction, that $|G \cap H| \geq l$, but $G \neq H$. By Assumption 3 we need to have $\gamma_G < \gamma_H$ or $\gamma_G > \gamma_H$; without loss of generality, assume the former. Then $H \succ G$ by Lemma 1, since $|G \cap H| \geq l$. Note that $G = G_q$ for some q and $H = G_j$ for some j such that $j < q$. Since H is stable, $\phi(G_j) = G_j$, but then $\mathcal{M}_q \neq \varnothing$ by (5), and so $\phi(G_q) \neq G_q$, as follows from (6). However, this contradicts the hypothesis that $G_q = G \in \mathcal{D}$, and thus completes the proof.

Part 2. By definition of mapping ϕ, $\phi(G_1) = G_1$, so $G_1 \in \mathcal{D}$. Take any government $G \in \mathcal{D}$; since $|G \cap G_1| \geq 0 = l$, we have $G = G_1$ by part 1. Consequently, $\mathcal{D} = \{G_1\}$, so $\mathcal{D}$ is a singleton. Now, for any G, $\phi(G) \in \mathcal{D}$, and thus $\phi(G) = G_1$.

Part 3. As before, the most competent government, G_1, is stable, i.e. $G_1 \in \mathcal{D}$. Now consider the set of governments which intersect with G_1 by fewer than l members:

$$\mathcal{B} = \{G \in \mathcal{G} : |G \cap G_1| < l\}.$$

This set is non-empty, because $n > 2k$ implies that there exists a government which does not intersect with G_1; obviously, it is in $\mathcal{B}$. Now take the most competent government from $\mathcal{B}$, G_j

where

$$j = \min\{q : 1 \le q \le |G| \text{ and } G_q \in \mathcal{B}\}.$$

We have $G_j \neq G_1$, because $G_1 \notin \mathcal{B}$. Let us show that G_j is stable. Note that any government G_q such that $\gamma_{G_q} > \gamma_{G_j}$ does not belong to $\mathcal{B}$ and therefore has at least l common members with stable government G_1. Hence, $\phi(G_q) = G_1$ (see (6)), and therefore G_q is unstable, except for the case $q = 1$. Now we observe that set $\mathcal{M}_j$ is empty: for each government G_q with $1 < q \le j$ either the first condition in (8) is violated (if $q = 1$) or the second one (otherwise). But this implies that $\phi(G_j) = G_j$, so G_j is stable. This proves that if $l \ge 1$, $\mathcal{D}$ contains at least two elements. Finally, note that this boundary is achieved: for example, if $l = 1$ and $n < 3k$.

Part 4. If $l = k$, then for any $G_q \in \mathcal{G}$, it is impossible that $H \succ G_q$ for some alternative $H \neq G_q$, as there will exist player $i \in G_q \setminus H$ for whom $w_i(H) < w_i(G_q)$. Hence, $\mathcal{M}_q = \varnothing$ for all q, and thus $\phi(G_q) = G_q$ by (6). Consequently, $\mathcal{D} = \mathcal{G}$, and this completes the proof of Proposition 1. ■

Proof of Proposition 2. Part 1. We prove the more general part 2, then the statement of part 1 will be a corollary: to obtain (11), one only needs to substitute $x = k$ into (12).

Part 2. Let us prove the existence of such stable government. Define a set-valued function $\chi : \mathcal{C}^l \to \mathcal{C}^{k-l} \cup \{\varnothing\}$ by

$$\chi(S) = \begin{cases} G \setminus S & \text{if } G \in \mathcal{D} \text{ and } S \subset G; \\ \varnothing & \text{if there exists no } G \in \mathcal{D} \text{ such that } S \subset G. \end{cases} \tag{A2}$$

In words, for any coalition of l individuals, function χ assigns a coalition of $k-l$ individuals such that their union is a stable government whenever such other coalition exists or an empty set when it does not exist. Note that $\chi(S)$ is a well defined single valued function: indeed, there cannot be two different stable governments G and H which contain S, for this would violate Proposition 1 (part 1), as they intersect by at least l members from S.

Let Y_{l-1} be some coalition of $l-1$ individuals such that $X \cap Y_{l-1} = \varnothing$; denote these individuals by $i_1, \dots, i_{l-1}$. We will now add $k-l+1$ individuals $i_l, \dots, i_k$ to this coalition one by one and we will denote the intermediate coalitions by $Y_l, \dots, Y_k$, and then prove that Y_k satisfies the requirements. Let $X_{l-1} = X$, and let

$$X_l = (X \cup Y_{l-1}) \cup \left(\bigcup_{i \in X} \chi(Y_{l-1} \cup \{i\}) \right). \tag{A3}$$

Intuitively, we take the set of individuals which are either forbidden to join the government under construction by our requirements (X) or are already there (Y_{l-1}), and add all individuals which can be in the same government with all individuals from Y_{l-1} and at least one individual from $X_{l-1} = X$. Now take some individual $i_l \in \mathcal{I} \setminus X_l$ (below we show that such individual exists) and let $Y_l = Y_{l-1} \cup \{i_l\}$. At each subsequent step z, $l+1 \leq z \leq k$, we choose zth individual for the government under construction as follows. We first define

$$X_z = (X \cup Y_{z-1}) \cup \left(\bigcup_{S \subset Y_{z-1}: |S| = l-1; i \in X} \chi (S \cup \{i\}) \right) \tag{A4}$$

and then take

$$i_z \in \mathcal{I} \setminus X_z \tag{A5}$$

(we prove that we can do that later) and denote $Y_z = Y_{z-1} \cup \{i_z\}$. Let the last government obtained in this way be denoted by $Y = Y_k$.

We now show that $\phi(Y) \cap X = \varnothing$. Suppose not, then there is individual $i \in \phi(Y) \cap X$. By (6) we must have that $|\phi(Y) \cap Y| \geq l$; take the individual i_j with the highest j of such individuals. Clearly, $j \geq l$, so individual i_j could not be a member of the initial Y_{l-1} and was added at a later stage. Now let S be a subset of $(\phi(Y) \cap Y) \setminus \{i_j\}$ such that $|S| = l-1$. Since government $\phi(Y)$ is stable and contains the entire S as well as $i \in X$ (and $i \notin S$ because $S \subset Y$ and $X \cap Y = \varnothing$), we must have $\chi(S \cup \{i\}) = \phi(Y)$. Consequently, if we consider the right-hand side of (A4) for $z = j$, we will immediately get that $\phi(Y) \subset X_j$, and therefore $i_j \in X_j$. But we picked i_j such that $i_j \in \mathcal{I} \setminus X_j$, according to (A5). We get to a contradiction, which proves that $\phi(Y) \cap X = \varnothing$, so $\phi(Y)$ is a stable government which contains no member of X.

It remains to show that we can always pick such individual; we need to show that the number of individuals in X_z is less than n for any $z : l \leq z \leq k$. Note that the union in the inner parentheses of (A4) consists of at most

$$(k-l)\binom{z-1}{l-1}x \leq (k-l)\binom{k-1}{l-1}x$$

individuals, while $z - 1 \leq k - 1$. Therefore, it is sufficient to require that

$$\begin{aligned} n &> x + k - 1 + (k-l)\binom{k-1}{l-1}x \\ &= x + k - 1 + x(k-l)\frac{(k-1)!}{(l-1)!(k-l)!}. \end{aligned}$$

Because we are dealing with integers, this implies (12), which completes the proof.

Part 3. This follows immediately by using Assumption 4 and setting $\rho = x$ in (12), which gives (13). ∎

Proof of Proposition 3. Part 1. By Assumption 2, $0 \leq l \leq k$, so either $l = 0$ or $l = 1$. If $l = 0$, then Proposition 1 (part 2) implies that the only stable government is G_1, so $\phi(G) = G_1$ for all $G \in \mathcal{G}$, where $G_1 = \{i_1\}$. If $l = 1$, then Proposition 1 (part 4) implies that any G is stable.

Part 2. In this case, either $l = 0$, $l = 1$, or $l = 2$. If $l = 0$ or $l = 2$, the proof is similar to that of part 1 and follows from Proposition 1 (parts 2 and 4). If $l = 1$, then $\{i_1, i_2\}$ is the most competent, and hence stable, government. By 1 (part 1), any other government containing i_1 or i_2 is unstable. Hence, $\{i_3, i_4\}$, the most competent government not containing i_1 or i_2, is stable. Proceeding likewise, we find that the only stable governments are $\{i_{2j-1}, i_{2j}\}$ for $1 \leq j \leq n/2$.

By the construction of mapping ϕ, either $\phi(G) = G$ or $|\phi(G) \cap G| = 1$. If $G = \{i_a, i_b\}$ with $a < b$, then $\phi(G)$ will include either i_a or i_b. Now it is evident that $\phi(G)$ will be the stable state which includes i_a, because it is more competent than the one which includes i_b if the latter exists and is different. ■

Proof of Theorem 3. The proof follows immediately from Theorem 1 and the assumption that changes are sufficiently infrequent. Indeed, in the latter case, all the strict inequalities in Definitions 1 and 2 are preserved. ■

Proof of Proposition 4. Part 1. If $l = 0$, then by Proposition 1 for any G, $\phi^t(G) = G_1^t$, where G_1^t is the most competent government $\{i_1^t, \ldots, i_k^t\}$.

Part 2. Suppose $l = 1$, then Proposition 1 provides a full characterization. There are $\lfloor n/k \rfloor \leq n/k$ stable governments. Each consists of k individuals, so the probability that a random new government coincides with any given stable government is $1/\binom{n}{k} = \frac{k!(n-k)!}{n!}$. The probability that it coincides with any stable government is $\lfloor n/k \rfloor / \binom{n}{k} \leq \frac{n}{k}\frac{k!(n-k)!}{n!} = \frac{(k-1)!(n-k)!}{(n-1)!} = 1/\binom{n-1}{k-1}$. The government will change to a more competent one if and only if it is unstable, which happens with probability greater than or equal to $1 - 1/\binom{n-1}{k-1}$.

The most competent government will be installed if and only if after the shock, the government contains at least 1 of the k most competent members. The probability that it does not contain any of these equals $\binom{n-k}{k}/\binom{n}{k}$ (this is the number of combinations that do not include k most competent members divided by the total number of combinations). We have

$$\begin{aligned}\binom{n-k}{k}\Big/\binom{n}{k} &= \frac{(n-k)!k!\,(n-k!)}{k!\,(n-2k)!n!} \\ &= \frac{(n-k)!}{(n-2k)!}\,\frac{(n-k!)}{n!} \\ &= \prod_{j=1}^{k} \frac{n-k-j}{n-j}.\end{aligned}$$

Since each of the k factors tends to 1 as $n \to \infty$, so does the product. Hence, the probability that the most competent government will arise, $\pi_t(l, k, n \mid G, \{\Gamma_G\}) = 1 - \binom{n-k}{k}/\binom{n}{k}$, tends to 0 as $n \to \infty$.

Part 3. If $l = k$, then $\phi^t(G) = G$ for any t and G. Hence, the government will not change. It will be the most competent if it contains k most competent individuals, which happens with probability $1/\binom{n}{k}$. This is less than 1, which is the corresponding probability for

$l = 0$, so $\pi_t (l = k, k, n \mid G, \{\Gamma_G\}) < \pi_t (l = 0, k, n \mid G, \{\Gamma_G\})$. If $k \geq 2$, it is also less than the corresponding probability for $l = 1$: in the latter case, there are at least two governments which will lead to the most competent one: $\{i_1^t, \ldots, i_k^t\}$ and $\{i_1^t, \ldots, i_{k-1}^t, i_{k+1}^t\}$, i.e., $\pi_t (l = 0, k, n \mid G, \{\Gamma_G\}) \geq 2/\binom{n}{k} > \pi_t (l = 0, k, n \mid G, \{\Gamma_G\})$. This completes the proof. ■

Proof of Proposition 5. Given the specific changes in $\{\Gamma_G\}$ in this case, the probability of having the most competent government G_1^t (for any initial G and $\{\Gamma_G\}$) is the probability that at least l members of G^t are members of G_1^t. This probability equals (from hypergeometric distribution):

$$\pi_t (l, k, n \mid G, \{\Gamma_G\}) = \frac{\sum_{q=l}^{k} \binom{k}{q} \binom{n-k}{k-q}}{\binom{n}{k}},$$

and is strictly decreasing in l. ■

Proof of Proposition 6. Part 1. Any such swapping (or, more generally, any transposition σ, where $\sigma(i)$ is the individual whose former competence individual i now has) induces a one-to-one mapping that maps government G to government $\rho(G)$: $i \in \rho(G)$ if and only if $\sigma(i) \in G$. By construction, $\Gamma_G^{t-1} = \Gamma_{\rho(G)}^t$, and, by construction of mapping ϕ, $\phi^{t-1}(G) = \phi^t(\rho(G))$ for all G. If all transitions occur in one stage, and a shock triggers a period of instability, then with probability 1 all shocks arrive at times t where government G^{t-1} is ϕ^{t-1}-stable.

If abilities of only two individuals are swapped, then $|G \cap \rho(G)| \geq k - 1 \geq l$. But G is ϕ^{t-1}-stable with probability 1, hence, $\rho(G)$ is ϕ^t-stable. Consider two cases. If $\Gamma_G^t \geq \Gamma_{\rho(G)}^t$, then $\Gamma_{\phi^t(G)}^t \geq \Gamma_G^t \geq \Gamma_{\rho(G)}^t$. If $\Gamma_G^t < \Gamma_{\rho(G)}^t$, then again $\Gamma_{\phi^t(G)}^t \geq \Gamma_{\rho(G)}^t$, since there is a ϕ^t-stable government $\rho(G)$ which has with G at least l common members and the competence of which is $\Gamma_{\rho(G)}^t$. Hence, $\phi^t(G)$ is either $\rho(G)$ or a more competent government. Hence, the competence of government cannot decrease. However, it may increase, unless G contains k most competent members. Indeed, in that case there exist $i, j \in \mathcal{I}$ with $i < j$ such that $i \notin G$ and $j \in G$. Obviously, swapping the abilities of these individuals increases the competence of G: $\Gamma_G^t > \Gamma_G^{t-1}$, and thus the stable government that will evolve will satisfy $\Gamma_{\phi(G)}^t \geq \Gamma_G^t > \Gamma_G^{t-1}$. Since the probability of this swapping is non-zero, eventually the competence of government will improve. Since there is a finite number of possible values of current government's competence, then with probability 1 the most competent government will emerge.

Part 2. This follows from an argument of part 1, taking into account that if abilities of x individuals changed, then $|G \cap \rho(G)| \geq k - \lfloor x/2 \rfloor \geq l$. Indeed, if $|G \cap \rho(G)| < k - \lfloor x/2 \rfloor$, we would have $|G \cap \rho(G)| \leq k - \lfloor (x+1)/2 \rfloor$ since the numbers of both sides are integers, and thus $|(G \setminus \rho(G)) \cup (\rho(G) \setminus G)| > 2 \lfloor (x+1)/2 \rfloor \geq x$. However, all individuals in $(G \setminus \rho(G)) \cup (\rho(G) \setminus G)$ changed their abilities, so the last inequality contradicts the assumption that no more than x individuals did. This contradiction completes the proof. ■

Proof of Proposition 7. The probability of having the most able player in the government under the royalty system is 1. Indeed, government $\phi^t\left(G^t\right)$ will for any t and any G^t consist of l irreplaceable members and $k-l$ most competent members. Since $l<k$, this always includes the most competent player. In the case of a junta-like system, there is a positive probability that a government that does not include player i_1^t is stable. If $\frac{a_1-a_2}{a_2-a_n}$ is sufficiently large, any government that includes player i_1^t is more competent that a government that does not. The first part follows. Now consider the probability that the least competent player, i_n^t, is a part of the government. In a royalty system, this can happen if and only if the irreplaceable member is the least competent, i.e., with probability $1/n$. In a junta-like system, the probability that the most competent government is installed is the probability that one of the players $i_1^t, \ldots, i_k^t$ is in the government immediately after the shock. This probability is higher than the probability that any given player is among $i_1^t, \ldots, i_k^t$, which is $k/n \geq 2/n$. This completes the proof. ■

Appendix B

This Appendix presents an extensive-form game in which individuals make proposals for alternative governments and vote over alternatives. It defines the Markov perfect equilibria (MPE) of this game, establishes their existence and their properties, and shows the equivalence between the notions of MPE and that of the (stochastic) political equilibrium defined in the text. We also provide a number of examples referred to in the text (e.g., on possibility of cyclic MPE or political equilibria, and on changes in expected competence).

Dynamic Game

We now present a dynamic game that captures certain salient features of the process of government change. This game involves different individuals proposing alternative governments and then all individuals (including current members of the government) voting over these proposals. For reasons that will become clear below, the game also involves an additional state variable, which governs when new proposals and transitions can be made.

Let us first introduce this additional state variable, denoted by v^t, which determines whether the current government can be changed. In particular, v^t takes two values: $v^t = s$ corresponds to a "sheltered" political situation (or "stable" political situation, although we reserve the term "stable" for governments which persist over time) and $v^t = u$ designates an unstable situation. The government can only be changed during unstable times. A sheltered political situation destabilizes (becomes unstable) with probability r in each period, that is, $\mathbf{P}\left(v^t = u \mid v^{t-1} = s\right) = r$. These events are independent across periods and we also assume that $v^0 = u$. An unstable situation becomes sheltered when an incumbent government survives a challenge or is not challenged at all (as explained below).

We next describe the procedure for challenging an incumbent government. We start with some government G^t at time t. If at time t the situation is unstable, then all individuals $i \in \mathcal{I}$ are ordered according to some sequence η_{G^t}. Individuals, in turn, nominate subsets of alternative governments $A_i^t \subset \mathcal{G} \setminus \left\{G^t\right\}$ that will be part of the primaries. An individual may choose not to nominate any alternative government, in which case he may choose $A_i^t = \varnothing$. All nominated governments (except the incumbent) make up the set $\mathcal{A}^t$, so

$$\mathcal{A}^t = \left\{G \in \mathcal{G} \setminus \left\{G^t\right\} : G \in A_i \text{ for some } i \in \mathcal{I}\right\}. \tag{A1}$$

If $\mathcal{A}^t \neq \varnothing$, then all alternatives in $\mathcal{A}^t$ take part in the *primaries* at time t. The primaries work as follows. All of the alternatives in $\mathcal{A}^t$ are ordered $\xi_{G^t}^{\mathcal{A}^t}(1), \xi_{G^t}^{\mathcal{A}^t}(2), \ldots, \xi_{G^t}^{\mathcal{A}^t}\left(\left|\mathcal{A}^t\right|\right)$ according to some pre-specified order (depending on $\mathcal{A}^t$ and the current government G^t). We refer to this order as the *protocol*, $\xi_{G^t}^{\mathcal{A}^t}$. The primaries are then used to determine the challenging government

$G' \in \mathcal{A}^t$. In particular, we start with G'_1 given by the first element of the protocol $\xi^{\mathcal{A}^t}_{G^t}$, $\xi^{\mathcal{A}^t}_{G^t}(1)$. At the second step, G'_1 is voted against the second element, $\xi^{\mathcal{A}^t}_{G^t}(2)$. We assume that all votings are sequential (and show in the Appendix that the sequence in which votes take place does not have any affect on the outcome). If more than $n/2$ of individuals support the latter, then $G'_2 = \xi^{\mathcal{A}^t}_{G^t}(2)$; otherwise $G'_2 = G'_1$. Proceeding in order, G'_3, G'_4, ..., and $G'_{|\mathcal{A}^t|}$ are determined, and G' is equal to the last element of the sequence, $G'_{|\mathcal{A}^t|}$. This ends the primary.

After the primary, the challenger G' is voted against the incumbent government G^t. G' wins if and only if a winning coalition of individuals (i.e., a coalition that belongs to $\mathcal{W}^t_{G^t}$) supports G'. Otherwise, we say that the incumbent government G^t wins. If $\mathcal{A}^t = \varnothing$ to start with, then there is no challenger and the incumbent government is again the winner.

If the incumbent government wins, it stays in power, and moreover the political situation becomes sheltered, that is, $G^{t+1} = G^t$ and $v^{t+1} = s$. Otherwise, the challenger becomes the new government, but the situation remains unstable, that is, $G^{t+1} = G'$ and $v^{t+1} = v^t = u$. All individuals receive stage payoff $w_i\left(G^t\right)$ (we assume that the new government starts acting from the next period on).

More formally, the exact procedure is as follows.

- Period $t = 0, 1, 2, \ldots$ begins with government G^t in power. If the political situation is sheltered, $v^t = s$, then each individual $i \in \mathcal{I}$ receives stage payoff $u_i^t\left(G^t\right)$; in the next period, $G^{t+1} = G^t$, $v^{t+1} = v^t = s$ with probability $1 - r$ and $v^{t+1} = u$ with probability r.

- If the political situation is unstable, $v_t = u$, then the following events take place:

  1. Individuals are ordered according to η_{G^t}, and in this sequence, each individual i nominates a subset of feasible governments $A_i^t \subset \mathcal{G} \setminus \left\{G^t\right\}$ for the primaries. These determine the set of alternatives $\mathcal{A}^t$ as in (A1).

  2. If $\mathcal{A}^t = \varnothing$, then we say that the incumbent government wins, $G^{t+1} = G^t$, $v^{t+1} = s$, and each individual receives states payoff $u_i^t\left(G^t\right)$. If $\mathcal{A}^t \neq \varnothing$, then the alternatives in $\mathcal{A}^t$ are ordered according to protocol $\xi^{\mathcal{A}^t}_{G^t}$.

  3. If $\mathcal{A}^t \neq \varnothing$, then the alternatives in $\mathcal{A}^t$ are voted against each other. In particular, at the first step, $G'_1 = \xi^{\mathcal{A}^t}_{G^t}(1)$. If $\left|\mathcal{A}^t\right| > 1$, then for $2 \leq j \leq \left|\mathcal{A}^t\right|$, at step j, alternative G'_{j-1} is voted against $\xi^{\mathcal{A}^t}_{G^t}(j)$. Voting in the primary takes place as follows: all individuals vote *yes* or *no* sequentially according to some pre-specified order, and $G'_j = \xi^{\mathcal{A}^t}_{G^t}(j)$ if and only if the set of the individuals who voted *yes*, $\mathcal{Y}_j^t$, is a simple majority (i.e., if $\left|\mathcal{Y}_j^t\right| > n/2$); otherwise, $G'_j = G'_{j-1}$. The challenger is determined as $G' = G'_{|\mathcal{A}^t|}$.

4. Government G' challenges the incumbent government G^t, and voting in the election takes place. In particular, all individuals vote *yes* or *no* sequentially according to some pre-specified order, and G' wins if and only if the set of the individuals who voted *yes*, $\mathcal{Y}^t$, is a winning coalition in G^t (i.e., if $\mathcal{Y}^t \in \mathcal{W}^t_{G^t}$); otherwise, G^t wins.
5. If G^t wins, then $G^{t+1} = G^t$, $v^{t+1} = s$; if G' wins, then $G^{t+1} = G'$, $v^{t+1} = u$. In either case, and each individual obtains stage payoff $u^t_i\left(G^t\right)$.

There are several important features about this dynamic game that are worth emphasizing. First, the set of winning coalitions, $\mathcal{W}^t_{G^t}$ when the government is G^t, determines which proposals for governmental change are accepted. Second, to specify a well defined game we had to introduce the pre-specified order η_G in which individuals nominate alternatives for the primaries, the protocol $\xi^{A^t}_G$ for the order in which alternatives are considered, and also the order in which votes are cast. Ideally we would like these orders not to have a major influence on the structure of equilibria, since they are not an essential part of the economic environment and we do not have a good way of mapping the specific orders to reality. We will see that this is indeed the case in the equilibria of interest. Finally, the rate at which political situations become unstable, r, has an important influence on payoffs by determining the rate at which opportunities to change the government arise. In what follows, we assume that r is relatively small, so that political situations are not unstable most of the time. Here, it is also important that political instability ceases after the incumbent government withstands a challenge (or if there is no challenge). This can be interpreted as the government having survived a "no-confidence" motion. In addition, as in the text, we focus on situations in which the discount factor β is large.

Strategies and Definition of Equilibrium

We define strategies and equilibria in the usual fashion. In particular, let h^{t,Q^t} denote the history of the game up to period t and stage Q^t in period t (there are several stages in period t if $v^t = u$). This history includes all governments, all proposals, votes, and stochastic events up to this time. The set of histories is denoted by $\mathcal{H}^{t,Q^t}$. A history h^{t,Q^t} can also be decomposed into two parts. We can write $h^{t,Q^t} = \left(h^t, Q^t\right)$ and correspondingly, $\mathcal{H}^{t,Q^t} = \mathcal{H}^t \times \mathcal{Q}^t$, where h^t summarizes all events that have taken place up to period $t-1$ and Q^t is the list of events that have taken place within the time instant t when there is an opportunity to change the government.

A strategy for individual $i \in \mathcal{I}$, denoted by σ_i, maps $\mathcal{H}^{t,Q^t}$ (for all t and Q^t) into a proposal when i nominates an alternative government (i.e., at the first stage of the period where $v^t = u$) and a vote for each possible proposal at each possible decision node (recall that the ordering of alternatives is automatic and is done according to a protocol). A Subgame Perfect Equilibrium (SPE) is a strategy profile $\{\sigma_i\}_{i\in\mathcal{I}}$ such that the strategy of each i is the best response to the strategies of all other individuals for all histories. Since there can be several SPE in dynamic games, many supported by complex trigger strategies, which are not our focus here, in this Appendix, we will limit our attention to the Markovian subset of SPEs. We next introduce the standard definition of Markov Perfect Equilibrium (MPE) in pure strategies:

Definition 3 *A* Markov Perfect Equilibrium *is an SPE profile of strategies* $\{\sigma_i^*\}_{i \in \mathcal{I}}$ *such that* σ_i^* *for each* i *in each period* t *depends only on* G^t, Γ^t, $\mathcal{W}^t$, *and* Q^t *(previous actions taken in period* t*).*

MPEs are natural in such dynamic games, since they enable individuals to condition on all of the payoff-relevant information, but rule out complicated trigger-like strategies, which are not our focus in this paper. It turns out that even MPEs potentially lead to a very rich set of behavior. For this reason, it is also useful to consider subsets of MPEs, in particular, *acyclic MPEs* and *order-independent MPEs*. As discussed in the text, an equilibrium is acyclic if cycles (changing the initial government but then reinstalling it at some future date) do not take place along the equilibrium path. Cyclical MPEs are both less realistic and also more difficult to characterize, motivating our main focus on acyclic MPEs. Formally, we have:

Definition 4 *An MPE* σ^* *is* cyclic *if the probability that there exist* $t_1 < t_2 < t_3$ *such that* $G^{t_3} = G^{t_1} \neq G^{t_2}$ *along the equilibrium path is positive. An MPE* σ^* *is* acyclic *if it is not cyclic.*

Another relevant subset of MPEs, order-independent MPEs or simply order-independent equilibria, is introduced by Moldovanu and Winter (1995). These equilibria require that strategies should not depend on the order in which certain events (e.g., proposal-making) unfold. Here we generalize (and slightly modify) their definition for our present context. For this purpose, let us denote the above-described game when the set of protocols is given by $\xi = \left\{\xi_G^{\mathcal{A}^t}\right\}_{G \in \mathcal{G}, \mathcal{A}^t \in \mathcal{P}(\mathcal{G}), G \notin \mathcal{A}^t}$ as $GAME[\xi]$ and denote the set of feasible protocols by $\mathcal{X}$.

Definition 5 *Consider* $GAME[\xi]$. *Then* σ^* *is an* order-independent equilibrium *for* $GAME[\xi]$ *if for any* $\xi' \in \mathcal{X}$, *there exists an equilibrium* σ'^* *of* $GAME[\xi']$ *such that* σ^* *and* σ'^* *lead to the same distributions of equilibrium governments* $G^\tau \mid G^t$ *for* $\tau > t$.

We will establish the relationship between acyclic and order-independent equilibria in Theorem 5.[19]

[19] One could also require order independence with respect to η as well as with respect to π. It can be easily verified that the equilibria we focus on already satisfy this property and hence, this is not added as a requirement of "order independence" in Definition 5.

Characterization of Markov Perfect Equilibria

Recall the mapping defined by $\phi : \mathcal{G} \to \mathcal{G}$ be the mapping defined by (6). We use the next theorem to establish the equivalence between political equilibria and MPE in the dynamic game.

Theorem 4 *Consider the game described above. Suppose that Assumptions 1-3 hold and let $\phi : \mathcal{G} \to \mathcal{G}$ be the political equilibrium given by (6). Then there exists $\varepsilon > 0$ such that if $\beta < 1 - \varepsilon$ and $r/(1-\beta) > \varepsilon$ then for any protocol $\xi \in \mathcal{X}$:*

1. *There exists an acyclic MPE in pure strategies σ^*.*
2. *Take an acyclic MPE in pure or mixed strategies σ^*. Then we have that*
   - *if $\phi(G^0) = G^0$, then there are no transitions;*
   - *otherwise, with probability 1 there exists a period t where the government $\phi(G^0)$ is proposed, wins the primaries, and wins the power struggle against G^t. After that, there are no transitions, so $G^\tau = \phi(G^0)$ for all $\tau \geq t$.*

Proof of Theorem 4. Part 1. The proof of this theorem relies on Lemma 1 introduced in Appendix A. We take β_0 such that for any $\beta > \beta_0$ the following inequalities are satisfied:

$$\begin{aligned} \text{for any } G, G', H, H' &\in \mathcal{G} \text{ and } i \in \mathcal{I} : w_i(G) < w_i(H) \text{ implies} \\ \left(1 - \beta^{|\mathcal{G}|}\right) w_i(G') + \beta^{|\mathcal{G}|} w_i(G) &< \left(1 - \beta^{|\mathcal{G}|}\right) w_i(H') + \beta^{|\mathcal{G}|} w_i(H). \end{aligned} \tag{A2}$$

For each $G \in \mathcal{G}$, define the following mapping $\chi_G : \mathcal{G} \to \mathcal{G}$:

$$\chi_G(H) = \begin{cases} \phi(H) & \text{if } H \neq G \\ G & \text{if } H = G \end{cases}.$$

Take any protocol $\xi \in \mathcal{X}$. Now take some node of the game in the beginning of some period t when $\nu^t = u$. Consider the stages of the dynamic game that take place in this period as a finite game by assigning the following payoffs to the terminal nodes:

$$v_i(G, H) = \begin{cases} w_i(H) + \frac{\beta}{1-\beta} w_i(\phi(H)) & \text{if } H \neq G \\ \frac{1+r\beta}{1-\beta(1-r)} w_i(G) + \frac{r\beta^2}{(1-\beta)(1-\beta(1-r))} w_i(\phi(G)) & \text{if } H = G \end{cases}, \tag{A3}$$

where $H = G^{t+1}$ is the government that is scheduled to be in power in period $t+1$, i.e., the government that defeated the incumbent G^t if it was defeated and G^t itself if it was not. For any such period t, take a SPE in pure strategies $\sigma^*_G = \sigma^*_{G^t}$ of the truncated game, such that this SPE is the same for any two nodes with the same incumbent government; the latter requirement ensures that once we map these SPEs to a strategy profile σ^* of the entire game $GAME[\xi]$, this profile will be Markovian. In what follows, we prove that for any $G \in \mathcal{G}$, (a) if σ^*_G is played, then there is no transition if $\phi(G) = G$ and there is a transition to $\phi(G)$ otherwise and (b) actions in profile σ^* are best responses if continuation payoffs are taken from profile σ^* rather than assumed to be given by (A3). These two results will complete the proof of part 1.

We start with part (a); take any government G and consider the SPE of the truncated game σ_G^*. First, consider the subgame where some alternative H has won the primaries and challenges the incumbent government G. Clearly, proposal H will be accepted if and only if $\phi(H) \succ G$. This implies, in particular, from the construction of mapping ϕ, that if $\phi(G) = G$, then no alternative H may be accepted. Second, consider the subgame where nominations have been made and the players are voting according to protocol $\xi_G^{\mathcal{A}}$. We prove that if $\phi(G) \in \mathcal{A}$, then $\phi(G)$ wins the primaries regardless of ξ (and subsequently wins against G, as $\phi(\phi(G)) = \phi(G) \succ G$. This is proved by backward induction: assuming that $\phi(G)$ has number q in the protocol, let us show that if it makes its way to jth round, where $q \leq j \leq |\mathcal{A}|$, then it will win this round. The base is evident: if $\phi(G)$ wins in the last round, players will get $v(G, \phi(G)) = \chi_G(\phi(G)) = \frac{1}{1-\beta} w(\phi(G))$ (we drop the subscript for player to refer to w and v as vectors of payoffs), while if it loses, they either get $v(G, H)$ for $H \neq \phi(G)$. Clearly, voting for $\phi(G)$ is better for a majority of the population, and thus $\phi(G)$ wins the primaries and defeats G. The step is proven similarly, hence, in the subgame which starts from qth round, $\phi(G)$ defeats the incumbent government. Since this holds irrespective of what happens in previous rounds, this concludes the second step. Third, consider the stage where nominations are made, and suppose, to obtain a contradiction, that $\phi(G)$ is not proposed. Then, in the equilibrium, players get a payoff vector $v(G, H)$, where $H \neq \phi(G)$. But then, clearly, any member of $\phi(G)$ has a profitable deviation, which is to nominate $\phi(G)$ instead of or in addition to what he is nominating in profile σ_G^*. Since in a SPE there should be no profitable deviations, this competes the proof of part (a).

Part (b) is straightforward. Suppose that the incumbent government is G. If some alternative H defeats government G, then from part (a), the payoffs that players get starting from next period are given by $\frac{1}{1-\beta} w_i(H)$ if $\phi(H) = H$ and $w_i(H) + \frac{\beta}{1-\beta} w_i(\phi(H))$ otherwise; in either case, the payoff is exactly equal to $v_i(G, H)$. If no alternative defeats government G, then $\nu^{t+1} = s$ (the situation becomes stable), and after that, government G stays until the situation becomes unstable, and government $\phi(G)$ is in power in all periods ever since; this again gives the payoff $\frac{1+r\beta}{1-\beta(1-r)} w_i(G) + \frac{r\beta^2}{(1-\beta)(1-\beta(1-r))} w_i(\phi(G))$. This implies that the continuation payoffs are indeed given by $v_i(G, H)$, which means that if in the entire game profile σ^* is played, no player has a profitable deviation. This proves part 1.

Part 2. Suppose σ^* is an acyclic MPE. Take any government $G = G^t$ at some period t in some node on or off the equilibrium path. Define binary relation $\rightarrow$ on set $\mathcal{G}$ as follows: $G \rightarrow H$ if and only if either $G = H$ and G has a positive probability of staying in power when $G^t = G$

and $\nu^t = u$, or $G \neq H$ and $G^{t+1} = H$ with positive probability if $G^t = G$ and $\nu^t = u$. Define another binary relation $\mapsto$ on $\mathcal{G}$ as follows: $G \mapsto H$ if any only if there exists a sequence (perhaps empty) of different governments $H_1, \ldots, H_q$ such that $G \rightarrow H_1 \rightarrow H_2 \rightarrow \cdots \rightarrow H_q = H$ and $H \rightarrow H$. In other words, $G \mapsto H$ if there is an on-equilibrium path that involves a sequence of transitions from G to H and stabilization of political situation at H. Now, since σ^* is an acyclic equilibrium, there is no sequence that contains at least two different governments $H_1, \ldots, H_q$ such that $H_1 \rightarrow H_2 \rightarrow \cdots \rightarrow H_q \rightarrow H_1$. Suppose that for at least one $G \in \mathcal{G}$, the set $\{H \in \mathcal{G} : G \mapsto H\}$ contains at least two elements. From acyclicity it is easy to derive the existence of government G with the following properties: $\{H \in \mathcal{G} : G \mapsto H\}$ contains at least two elements, but for any element H of this set, $\{H' \in \mathcal{G} : H \mapsto H'\}$ is a singleton.

Consider the restriction of profile σ^* on the part of the game where government G is in power, and call it σ^*_G. The way we picked G implies that some government may defeat G with a positive probability, and for any such government H the subsequent evolution prescribed by profile σ^* does not exhibit any uncertainty, and the political situation will stabilize at the unique government $H' \neq G$ (but perhaps $H' = H$) such that $H \mapsto H'$ in no more than $|\mathcal{G}| - 2$ steps. Given our assumption (A2) and the assumption that r is small, this implies that no player is indifferent between two terminal nodes of this period which ultimately lead to two different governments H'_1 and H'_2, or between one where G stays and one where it is overthrown. But players act sequentially, one at a time, which means that the last player to act on the equilibrium path when it is still possible to get different outcomes must mix, and therefore be indifferent. This contradiction proves that for any G, government H such that $G \mapsto H$ is well defined. Denote this government by $\psi(G)$.

To complete the proof, we must show that $\psi(G) = \phi(G)$ for all G. Suppose not; then, since $\psi(G) \succ G$ (otherwise G would not be defeated as players would prefer to stay in G), we must have that $\Gamma_{\phi(G)} > \Gamma_{\psi(G)}$. This implies that if some alternative H such that $H \mapsto \phi(G)$ is nominated, it must win the primaries; this is easily shown by backward induction. If no such alternative is nominated, then, since there is a player who prefers $\phi(G)$ to $\psi(G)$ (any member of $\phi(G)$ does), such player would be better off deviating and nominating $\psi(G)$. A deviation is not possible in equilibrium, so $\psi(G) = \phi(G)$ for all G. By construction of mapping ψ, this implies that there are no transitions if $G = \phi(G)$ and one or more transitions ultimately leading to government $\phi(G)$ otherwise. This completes the proof. ■

The most important result from this theorem is that acyclic MPE lead to equilibrium transitions given by the same mapping ϕ, defined in (6), which characterize political equilibria as defined in Definition 1. This result thus provides further justification for the notion of political equilibrium used in our main analysis.

The hypothesis that r is sufficiently small ensures that stable political situations are sufficiently "stable," so that if the government passes a "no-confidence" voting, it stays in power for some nontrivial amount of time. Such a requirement is important to ensure that an MPE in pure strategies exists (which in turn allows us to obtain a characterization of equilibria), and is related in spirit to the second requirement in part 2 of Definition 1. Example 7, which is presented next, illustrates the potential for nonexistence of pure strategy MPE without this assumption.

Example 7 Suppose that the society consists of five individuals $1, 2, 3, 4, 5$ ($n = 5$). Suppose each government consists of two members, so $k = 2$. There is "almost perfect" democracy ($l = 1$), and suppose $m = 3$. Assume

$$\Gamma_{\{i,j\}} = 30 - \min\{i,j\} - 5\max\{i,j\}.$$

Moreover, assume that all individuals care a lot about being in the government, and about the competence if they are not in the government; however, if an individual compares utility of being a member of two different governments, she is almost indifferent. In this environment, there are two fixed points of mapping ϕ: $\{1, 2\}$ and $\{3, 4\}$.

Let us show that there is no MPE in pure strategies if $v^t = u$ for all t (so that the incumbent government is contested in each period). Suppose that there is such equilibrium for some protocol ξ. One can easily see that no alternative may win if the incumbent government is $\{1, 2\}$: indeed, if in equilibrium there is a transition to some $G \neq \{1, 2\}$, then in the last voting, when $\{1, 2\}$ is challenged by G, both 1 and 2 would be better off rejecting the alternative and postponing transition to the government (or a chain of governments) that they like less. It is also not hard to show that any of the governments that include 1 or 2 (i.e., $\{1, 3\}$, $\{1, 4\}$, $\{1, 5\}$, $\{2, 3\}$, $\{2, 4\}$, and $\{2, 5\}$) lose the contest for power to $\{1, 2\}$ in equilibrium. Indeed, if $\{1, 2\}$ is included in the primaries, it must be the winner (intuitively, this happens because $\{1, 2\}$ is the Condorcet winner in simple majority votings). Given that, it must always be included in the primaries, for otherwise individual 1 would have a profitable deviation and nominate $\{1, 2\}$. We can now conclude that government $\{3, 4\}$ is stable: any government which includes 1 or 2 will immediately lead to $\{1, 2\}$ which is undesirable for both 3 and 4, while $\{3, 5\}$ and $\{4, 5\}$ are worse than $\{3, 4\}$ for 3 and 4 as well; therefore, if there is some transition in equilibrium, then 3 and 4 are better off staying at $\{3, 4\}$ for an extra period, which makes a profitable deviation.

We now consider the governments $\{3, 5\}$ and $\{4, 5\}$. First, we rule out the possibility that from $\{3, 5\}$ the individuals move to $\{4, 5\}$ and vice versa. Indeed, if this was the case, then in the last voting when the government is $\{3, 5\}$ and the alternative is $\{4, 5\}$, individuals $1, 2, 3, 5$ would be better off blocking this transition (i.e., postponing it for one period). Hence, either one of governments $\{3,5\}$ and $\{4,5\}$ is stable or one of them leads to $\{3,4\}$ in one step or $\{1,2\}$ in two steps. We consider these three possibilities for the government $\{3,5\}$ and arrive to a contradiction; the case of $\{4,5\}$ may be considered similarly and also leads to a contradiction.

It is trivial to see that a transition to $\{1,2\}$ (in one or two steps) cannot be an equilibrium. If this were the case, then in the last voting, individuals 3 and 5 would block this transition, since they are better off staying in $\{3,5\}$ for one more period (even if the intermediate step to $\{1,2\}$ is a government that includes either 3 or 5). This is a profitable deviation which cannot happen in an equilibrium. It is also trivial to check that $\{3,5\}$ cannot be stable. Indeed, if this was the case, then if alternative $\{3,4\}$ won the primaries, it would be accepted, as individuals $1,2,3,4$ would support it. At the same time, any alternative that would lead to $\{1,2\}$ would not be accepted, and neither will be alternative $\{4,5\}$, unless it leads to $\{3,4\}$. Because of that, alternative $\{3,4\}$ would make its way through the primaries if nominated, for it is better than $\{3,5\}$ for a simple majority of individuals. But then $\{3,4\}$ must be nominated, for, say, individual 4 is better off if it were, since he prefers $\{3,4\}$ to $\{3,5\}$. Consequently, if $\{3,5\}$ were stable, we would get a contradiction, since we proved that in this case, $\{3,4\}$ must be nominated, win the primaries and take over the incumbent government $\{3,5\}$.

The remaining case to consider is where from $\{3,5\}$ the individuals transit to $\{3,4\}$. Note that in this case, alternative $\{1,2\}$ would be accepted if it won the primaries: indeed, individuals 1 and 2 prefer $\{1,2\}$ over $\{3,4\}$ for obvious reasons, but individual 5 is also better off if $\{1,2\}$ is accepted, even if the former grants him an extra period of staying in power (as the discount factor β is close to 1). Similarly, any alternative which would lead to $\{1,2\}$ in the next period must also be accepted in the last voting. This implies, however, that such alternative ($\{1,2\}$ or some other one which leads to $\{1,2\}$) must necessarily win the primaries if nominated (by the previous discussion, $\{4,5\}$ may not be a stable government, and hence the only choice the individuals make is whether to move ultimately to $\{3,4\}$ or to $\{1,2\}$, of which they prefer the latter). This, in turn, means that $\{1,2\}$ must be nominated, for otherwise, say, individual 1 would be better off doing that. Hence, we have come to a contradiction in all possible cases, which proves that for no protocol ξ there exists a MPE in pure strategies. Thus, the proof that both cyclic and acyclic MPEs do not exist is complete.

Cycles, Acyclicity, and Order-Independent Equilibria

The acyclicity requirement in Theorem 4 (similar to the requirement of acyclic political equilibrium in Theorem 1 in the text) is not redundant. We next provide an example of a cyclic MPE.

Example 8 Consider a society consisting of five individuals ($n = 5$). The only feasible governments are $\{1\},\{2\},\{3\},\{4\},\{5\}$. Suppose that there is "perfect democracy," i.e. $l_G = l = 0$ for $G \in \mathcal{G}^k$, and that voting takes the form of a simple majority rule, i.e $m_G = m = 3$ for all G. Suppose also that the competences of different feasible governments are given by

$$\Gamma_{\{i\}} = 5 - i,$$

so $\{1\}$ is the best government.

Assume also that stage payoffs are given as in Example 2. In particular,

$$w_i(G) = \Gamma_G + 100\mathrm{I}_{\{i \in G\}}.$$

These utilities imply that each individual receives a high value from being part of the government relative to the utility she receives from government competence.

Finally, we define the protocols ξ_G^A as follows. If $G = \{1\}$, then $\xi_G^{\mathcal{G}\backslash\{G\}} = \xi_{\{1\}}^{\{\{2\},\{3\},\{4\},\{5\}\}} = (\{3\},\{4\},\{5\},\{2\})$ and $\xi_{\{1\}}^A$ for $A \neq (\{3\},\{4\},\{5\},\{2\})$ is obtained from $\xi_{\{1\}}^{\{2,3,4,5\}}$ by dropping governments which are not in A: for example, $\xi_{\{1\}}^{\{\{2\},\{3\},\{5\}\}} = (\{3\},\{5\},\{2\})$. For other governments, we define $\xi_{\{2\}}^{\{\{1\},\{3\},\{4\},\{5\}\}} = (\{4\},\{5\},\{1\},\{3\})$, $\xi_{\{3\}}^{\{\{1\},\{2\},\{4\},\{5\}\}} = (\{5\},\{1\},\{2\},\{4\})$, $\xi_{\{4\}}^{\{\{1\},\{2\},\{3\},\{5\}\}} = (\{1\},\{2\},\{3\},\{5\})$ and $\xi_{\{5\}}^{\{\{1\},\{2\},\{3\},\{4\}\}} = (\{2\},\{3\},\{4\},\{1\})$, and for other A again define ξ_G^A by dropping the governments absent in A. Then there exists an equilibrium where the governments follow a cycle of the form $\{5\} \to \{4\} \to \{3\} \to \{2\} \to \{1\} \to \{5\} \to \cdots$.

To verify this claim, consider the following nomination strategies by the individuals. If the government is $\{1\}$, two individuals nominate $\{2\}$ and the other three nominate $\{5\}$; if it is $\{2\}$, two individuals nominate $\{3\}$ and three nominate $\{1\}$; if it is $\{3\}$, two nominate $\{4\}$ and three nominate $\{2\}$; if it is $\{4\}$, two nominate $\{5\}$ and three nominate $\{3\}$; if it is $\{5\}$, two nominate $\{1\}$ and three nominate $\{4\}$.

Let us next turn to voting strategies. Here we appeal to Lemma 1 from Acemoglu, Egorov and Sonin (2008), which shows that in this class of games, it is sufficient to focus on strategies in which individuals always vote for the alternative yielding the highest payoff for them at each stage. In equilibrium, any alternative government which wins the primaries, on or off equilibrium path, subsequently wins against the incumbent government. In particular, in such an equilibrium supporting the incumbent government breaks a cycle, but only one person (the member of the incumbent government) is in favor of it. We next show if only one individual deviates at the nomination stage, then next government in the cycle still wins in the primaries. Suppose that the current government is $\{3\}$ (other cases are treated similarly). Then by construction, governments $\{2\}$ and $\{4\}$ are necessarily nominated, and $\{1\}$ or $\{5\}$ may either be nominated or not. If the last voting in the primaries is between $\{2\}$ and $\{4\}$, then $\{2\}$ wins: indeed, all individuals know that both alternatives can take over the incumbent government, but $\{2\}$ is preferred by individuals 1, 2, and 5 (because they would want to be government members earlier rather than later). If, however, the last stage involves voting between $\{4\}$ on the one hand and either $\{1\}$ or $\{5\}$ on the other, then $\{4\}$ wins for similar reason. Now, if either $\{1\}$ or $\{5\}$ is nominated, then in the first voting it is voted against $\{2\}$. All individuals know that accepting $\{2\}$ will ultimately lead to a transition to $\{2\}$, whereas supporting $\{1\}$ or $\{5\}$ will lead to $\{4\}$. Because of that, at least three individuals (1, 2, 5) will support $\{2\}$. This proves that $\{2\}$ will win against the incumbent government $\{3\}$, provided that $\{2\}$ and $\{4\}$ participate in the primaries, which is necessarily the case if no more than one individual deviates. This, in turn, implies that nomination strategies are also optimal in the sense that there is no profitable one-shot deviation for any individual. We can easily verify that this holds for other incumbent governments as well.

We have thus proved that the strategies we constructed form an SPE; since they are also Markovian, it is a MPE as well. Along the equilibrium path, the governments follow a cycle $\{5\} \to \{4\} \to \{3\} \to \{2\} \to \{1\} \to \{5\} \to \cdots$. We can similarly construct a cycle that moves in the other direction: $\{1\} \to \{2\} \to \{3\} \to \{4\} \to \{5\} \to \{1\} \to \cdots$ (though this would require different protocols). Hence, for some protocols, cyclic equilibria are possible.

Intuitively, a cycle enables different individuals that will not be part of the limiting (stable) government to enjoy the benefits of being in power. This example, and the intuition we suggest, also highlight that even when there is a cyclic equilibrium, an acyclic equilibrium still exists. (This is clear from the statement in Theorem 1, and also from Theorem 5). Example 8 also makes it clear that cyclic equilibria are somewhat artificial and less robust. Moreover, as emphasized in Theorems 1 and 4, acyclic equilibria have an intuitive and economically meaningful structure. In the text, we showed how certain natural restrictions rule out cyclic political equilibria (Theorem 2). Here we take a complementary approach and show that the refinement of MPE introduced above, *order independence*, is also sufficient to rule out cyclic equilibria (even without the conditions in Theorem 2). This is established in the next theorem, which also shows that with order-independent MPE, multi-step transitions, which are possible under MPE as shown in the next example, will also be ruled out.

Example 9 Take the setup of Example 8, with the exception that $l_{\{1\}} = 1$ (so that consent of individual 1 is needed to change the government when the government is $\{1\}$). It is then easy to check that the strategy profile constructed in Example 8 is a MPE in this case as well. However, since individual 1 will vote against any alternative which wins the primaries, the difference is that alternative $\{5\}$ will not be accepted in equilibrium and government $\{1\}$ will persist. Hence, in equilibrium, the transitions are as follows: $\{5\} \to \{4\} \to \{3\} \to \{2\} \to \{1\}$.

We now establish that order-independent equilibria always exist, are always acyclic, and lead to rapid (one-step) equilibrium transitions. As such, this theorem will be a strong complement to Theorem 2 in the text, though its proof requires a slightly stronger version of Assumption 3, which we now introduce.

Assumption $3'$ *For any $i \in \mathcal{I}$ and any sequence of feasible governments, $H_1, H_2, \ldots, H_q \in \mathcal{G}$ (for $q \geq 2$), we have*

$$w_i(H_1) \neq \frac{\sum_{j=2}^{q} w_i(H_j)}{q-1}.$$

Recall that Assumption 3 imposed that no two feasible governments have exactly the same competence. Assumption $3'$ strengthens this and requires that the competence of any government should not be the average of the competences of other feasible governments. Like Assumption 3, Assumption $3'$ is satisfied "generically," in the sense that if it were not satisfied for a society, any small perturbation of competence levels would restore it.

Theorem 5 *Consider the game described above. Suppose that Assumptions 1, 2 and $3'$ hold and let $\phi : \mathcal{G} \to \mathcal{G}$ be the political equilibrium defined by (6). Then there exists $\varepsilon > 0$ such that if $\beta < 1 - \varepsilon$ and $r/(1-\beta) > \varepsilon$ for any protocol $\xi \in \mathcal{X}$:*

1. *There exists an order-independent MPE in pure strategies σ^*.*
2. *Any order-independent MPE in pure strategies σ^* is acyclic.*
3. *In any order-independent MPE σ^*, we have that:*
   - *if $\phi(G^0) = G^0$, then there are no transitions and government $G^t = G^0$ for each t;*
   - *if $\phi(G^0) \neq G^0$, then there is a transition from G^0 to $\phi(G^0)$ in period $t = 0$, and there are no more transitions: $G^t = \phi(G^0)$ for all $t \geq 1$.*
4. *In any order-independent MPE σ^*, the payoff of each individual $i \in \mathcal{I}$ is given by*

$$u_i^0 = w_i(G^0) + \frac{\beta}{1-\beta} w_i(\phi(G^0)).$$

Proof of Theorem 5. Part 1. In part 1 of Theorem 4, we proved that for any $\xi \in \mathcal{X}$ there exists a MPE in pure strategies, and from part 2 of Theorem 4 it follows that these MPE constructed for different $\xi \in \mathcal{X}$ have the same equilibrium path of governments. The existence of an order-independent equilibrium follows.

Part 2. Suppose, to obtain a contradiction, that order-independent MPE in pure strategies σ^* is cyclic. Define mapping $\chi : \mathcal{G} \to \mathcal{G}$ as follows: $\chi(G) = H$ if for any node on equilibrium path which starts with government $G^t = G$ and $\nu^t = u$, the next government $G^{t+1} = H$. Since the equilibrium is in pure strategies, this mapping is well defined and unique. The assumption that equilibrium σ^* is acyclic implies that there is a sequence of pair-wise different governments $H_1, H_2, \ldots, H_q$ (where $q \geq 2$) such that $\chi(H_j) = H_{j+1}$ for $1 \leq j < q$ and $\chi(H_q) = H_1$. Without loss of generality, assume that H_2 has the least competence of all governments $H_1, H_2, \ldots, H_q$. If $q = 2$, then the cycle has two elements, of which H_2 is the worse government. However, this implies that H_2 cannot defeat H_1 even if it wins the primaries, since all players, except, perhaps, those in $H_2 \setminus H_1$, prefer H_1 to an eternal cycle of H_1 and H_2. This immediate contradiction implies that we only need to consider the case $q \geq 3$.

If $q \geq 3$, then, by the choice of H_2, $\Gamma_{H_1} > \Gamma_{H_2}$ and $\Gamma_{H_3} > \Gamma_{H_2}$. Without loss of generality, we may assume that the protocol is such that if the incumbent government is H_1, H_3 is put at the end (if H_3 is nominated); this is possible since σ^* is an order-independent equilibrium. By definition, we must have that proposal H_2 is nominated and accepted in this equilibrium along the equilibrium path.

Let us first prove that alternative H_3 will defeat the incumbent government H_1 if it wins the primaries. Consider a player i who would have weakly preferred H_2, the next equilibrium government, to win over H_1 if H_2 won the primaries; since H_2 defeats H_1 on the equilibrium path, such players must form a winning coalition in H_1. If $i \notin H_2$, then H_2 brings i the lowest utility of all governments in the cycle; hence, i would be willing to skip H_2; hence, such i would be strictly better off if H_3 defeated H_1. Now suppose $i \in H_2$. If, in addition, $i \in H_1$, then he prefers H_1 to H_2. Assume, to obtain a contradiction, that i weakly prefers that H_3 does not defeat H_1; it is then easy to see that since he prefers H_1 to H_2, he would strictly prefer H_2 not to defeat H_1 if H_2 won the primaries. The last case to consider is $i \in H_2$ and $i \notin H_1$. If β is sufficiently close to 1, then, as implied by Assumption $3'$, player i will either prefer that both H_2 and H_3 defeat H_1 or that none of them does. Consequently, all players who would support H_2 also support H_3, which proves that H_3 would be accepted if nominated.

Let us prove that in equilibrium H_3 is not nominated. Suppose the opposite, i.e., that H_3 is nominated. Then H_2 cannot win the primaries: indeed, in the last voting, H_2 must face H_3, and since, as we showed, only members of H_2 may prefer that H_2 rather than H_3 is the next government, H_3 must defeat H_2 in this voting. This means that in equilibrium H_3 is not nominated.

Consider, however, what would happen if all alternatives were nominated. Suppose that some government G then wins the primaries. It must necessarily be the case that G defeats H_1: indeed, if instead H_1 would stay in power, then $G \neq H_3$ (we know that H_3 would defeat H_1), and this implies that in the last voting of the primaries, H_3 would defeat G. Let us denote the continuation utility that player i gets if some government H comes to power as $v_i(H)$. If there is at least one player with $v_i(G) > v_i(H_2)$, then this player has a profitable deviation during nominations: he can nominate all alternatives and ensure that G wins the primaries and defeats H_1. Otherwise, if $v_i(G) \leq v_i(H_2)$ for all players, we must have that $v_i(G) < v_i(H_3)$ for a winning coalition of players, which again means that G cannot win the primaries. This contradiction proves that for the protocol we chose, H_2 cannot be the next government, and this implies that there are no cyclic order-independent equilibria in pure strategies.

Part 3. The proof is similar to the proof of part 2. We define mapping χ in the same way and choose government H such that $\chi(\chi(H)) \neq \chi(H)$, but $\chi(\chi(\chi(H))) = \chi(\chi(H))$. We then take a protocol which puts government $\chi(\chi(H))$ at the end whenever it is nominated and come to a similar contradiction.

Part 4. This follows straightforwardly from part 3, since the only transition may happen at period $t = 0$. ■

Stochastic Markov Perfect Equilibria

We next characterize the structure of (order-independent) stochastic MPE (that is, order-independent MPE in the presence of stochastic shocks) and establish the equivalence between order-independent (or acyclic) MPE and our notion of (acyclic stochastic) political equilibrium. Once again, the most important conclusion from this theorem is that MPE of the dynamic game discussed here under stochastic shocks lead to the same behavior as our notion of stochastic political equilibrium introduced in Definition 2.

Theorem 6 *Consider the above-described stochastic environment. Suppose that Assumptions 1, 2, 3′, and 4 hold. Let $\phi^t : \mathcal{G} \to \mathcal{G}$ be the political equilibrium defined by (6) for Γ^t_G. Then there exists $\varepsilon > 0$ such that if $\beta < 1 - \varepsilon$, $r/(1-\beta) > \varepsilon$ and $\delta > \varepsilon$ then for any protocol $\xi \in \mathcal{X}$, we have the following results.*

1. *There exists an order-independent MPE in pure strategies.*

2. *Suppose that between periods t_1 and t_2 there are no shocks. Then in any order-independent MPE in pure strategies, the following results hold:*

    - *if $\phi(G^{t_1}) = G^{t_1}$, then there are no transitions between t_1 and t_2;*
    - *if $\phi(G^{t_1}) \neq G^{t_1}$, then alternative $\phi(G^{t_1})$ is accepted during the first period of instability (after t_1).*

Proof of Theorem 6. Part 1. If δ is sufficiently small, then the possibility of shocks does not change the ordering of continuation utilities at the end of any period any for any player. Hence, the equilibrium constructed in the proof of part 1 of Theorem 4 proves this statement as well.

Part 2. If δ is sufficiently small, the proof of Theorem 5 (parts 2 and 3) may be applied here with minimal changes, which are omitted. ■

Examples

Example 10 This example shows that in stochastic environments, even though likelihood of the best government coming to power is higher under more democratic institutions, the expected competence of stable governments may be lower. Suppose $n = 9$, $k = 4$, $l_1 = 3$, $m = 5$. Let the individuals be denoted $1, 2, 3, 4, 5, 6, 7, 8, 9$, with decreasing ability. Namely, suppose that abilities of individuals $1, \ldots, 8$ are given by $\gamma_i = 2^{8-i}$, and $\gamma_9 = -10^6$. Then the 14 stable governments, in the order of decreasing competence, are given as:

1234	2358
1256	2367
1278	2457
1357	2468
1368	3456
1458	3478
1467	5678

(Note that this would be the list of stable governments for any decreasing sequence $\{\gamma_i\}_{i=1}^{9}$, except for that, say, $\Gamma_{\{1368\}}$ may become than $\Gamma_{\{1458\}}$.) Now consider the same parameters, but take $l_2 = 2$. Then there are three stable governments 1234, 1567, and 2589. For a random initial government, the probability that individual 9 will be a part of the stable government that evolves is $\frac{9}{126} = \frac{1}{16}$: of $\binom{9}{4} = 126$ feasible governments there are 9 governments that lead to 2589, which are 2589, 2689, 2789, 3589, 3689, 3789, 4589, 4689, and 4789. Clearly, the expected competence of government for $l_2 = 2$ is negative, whereas for $l_1 = 3$ it is positive, as no stable government includes the least competent individual 9.

Example 11 There are $n = 19$ players and 3 feasible governments: $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{7, 8, 9, 10, 11, 12, 13\}$, $C = \{13, 14, 15, 16, 17, 18, 19\}$ (so $\bar{k} = 7$). The discount factor is sufficiently close to 1, say, $\beta > 0.999$. The institutional parameters of these governments and players' utilities from them are given in the following table.

G	l_G	m_G	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A	0	10	90	90	90	90	90	90	90	60	60	30	30	30	30	30	30	30	30	30	45
B	0	11	60	20	20	20	20	20	80	80	80	80	80	80	80	20	20	20	20	20	20
C	0	12	10	10	10	10	10	10	10	10	10	10	10	10	70	70	70	70	70	70	70

The utilities are constructed as follows. Members of government A receive 90, non-members receive 30, except for 19, who receives 45. Members of B receive 80, except for 7 who receives 30, non-members receive 10, except for 1, 2, 3 who receive 60. Note that Assumption 1 holds, and that the condition $7 = \bar{k} < m \leq n - \bar{k} = 12$ is satisfied for all feasible governments. Moreover, $m_G > n/2$ for all governments, which means that any two winning coalitions intersect.

We claim that ϕ given by $\phi(A) = C$, $\phi(B) = A$, $\phi(C) = B$ is a (cyclic) political equilibrium. Let us first check property (ii) of Definition 1. The set of players with $V_i(C) > V_i(A)$ is $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ (as β is close to 1, the simplest way to check this condition for player i is to verify whether $w_i(A)$ is greater than or less than the average of $w_i(A)$, $w_i(B)$, $w_i(C)$; the case where these are equal deserves more detailed study, and is critical for this example). These ten players form a winning coalition in A. The set of players with $V_i(A) > V_i(B)$ is $\{2, 3, 4, 5, 6, 14, 15, 16, 17, 18, 19\}$; these eleven players form a winning coalition in B. The set of players with $V_i(B) > V_i(C)$ is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$; these twelve players form a winning coalition in C.

Let us now check condition (ii) of Definition 1. Suppose the current government is A; then the only H we need to consider is B. Indeed, if $H = C$ then $V_i(H) > V_i(\phi(A)) = V_i(C)$ is impossible for any player, and if $H = A$, then the $V_i(H) > V_i(\phi(A))$ cannot hold for a winning coalition of players, as the opposite inequality $V_i(\phi(A)) > V_i(A)$ holds for a winning coalition (condition (i)), and any two winning coalitions intersect in this example. But for $H = B$, condition (ii) of Definition 1 is also satisfied, as $V_i(B) > w_i(A) / (1 - \beta)$ holds for players $\{10, 11, 12, 13, 14, 15, 16, 17, 18\}$ only, which is not a winning coalition, as there are only nine players (we used the fact that player 19 has $V_i(A) > V_i(B)$, but $V_i(B) > w_i(A) / (1 - \beta)$ for β close to 1, as 45 is the average of 20 and 70). If the current government is B, then, as before, only government $H = C$ needs to be considered. But $V_i(C) > V_i(A)$ holds for ten players only, and this is not a winning coalition in B. Finally, if the current government is C, then again, only the case $H = A$ needs to be checked. But $V_i(A) > V_i(B)$ holds for only eleven players, and this is not a winning coalition in C. So, both conditions of Definition 1 are satisfied, and thus ϕ is a cyclic political equilibrium.

References

Acemoglu, Daron, Philippe Aghion, and Fabrizio Zilibotti (2006) "Distance to Frontier, Selection, and Economic Growth." *Journal of the European Economic Association*, 4(1), pp. 37-74.

Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin (2008) "Dynamics and Stability in Constitutions, Coalitions, and Clubs," NBER Working Paper No. 14239.

Acemoglu, Daron and James Robinson (2000) "Why Did The West Extend The Franchise? Democracy, Inequality, And Growth In Historical Perspective," *Quarterly Journal of Economics,* 115(4), 1167-1199.

Acemoglu, Daron and James Robinson (2006) *Economic Origins of Dictatorship and Democracy*, Cambridge University Press.

Acemoglu, Daron and James Robinson (2008) "Persistence of Power, Elites and Institutions," *American Economic Review*, 98(1), pp. 267-93.

Acemoglu, Daron, James Robinson, and Thierry Verdier (2004) "Kleptocracy and Divide-and-Rule," *Journal of the European Economic Association*, 2 (2-3), 162-192.

Acemoglu, Daron, Davide Ticchi, and Andrea Vindigni (2007) "Emergence and Persistence of Inefficient States," forthcoming *Journal of European Economic Association.*

Alfoneh, Ali (2008) "Ahmadinejad vs. the Technocrats," American Enterprise Institute Online, http://www.aei.org/outlook/27966.

Aghion, Philippe, Alberto Alesina, and Francesco Trebbi (2009) "Democracy, Technology and Growth," in *Institutions and Economic Performance*, edited by Elhanan Helpman, Harvard University Press, Cambridge.

Baker, Peter and Susan Glasser (2007) *Kremlin Rising: Vladimir Putin's Russia and the End of Revolution*, updated edition, Potomac Books, Virginia.

Banks, Jeffrey S., and Rangarajan K. Sundaram (1998) "Optimal Retention in Principal/Agent Models," *Journal of Economic Theory*, 82(2), 293-323.

Barbera, Salvador, and Matthew Jackson (2004) "Choosing How to Choose: Self-Stable Majority Rules and Constitutions," *Quarterly Journal of Economics*, 119(3): 1011-1048.

Barbera, Salvador, Michael Maschler, and Jonathan Shalev (2001) "Voting for Voters: A Model of the Electoral Evolution," *Games and Economic Behavior*, 37: 40-78.

Barro, Robert (1973) "The Control of Politicians: An Economic Model," *Public Choice*, 14, 19-42.

Barro, Robert (1996) "Democracy and Growth," *Journal of Economic Growth*, 1(1), 1-27.

Besley, Timothy (2005) "Political Selection," *Journal of Economic Perspectives*, 19(3), pages 43-60.

Besley, Timothy and Anne Case (1995) "Incumbent behavior: vote seeking, tax setting and yardstick competition," *American Economic Review,* 85(1), 25-45.
Besley, Timothy and Stephen Coate (1997) "An Economic Model of Representative Democracy," *Quarterly Journal of Economics,* vol. 112(1): 85-114.
Besley, Timothy and Stephen Coate (1998) "Sources of Inefficiency in a Representative Democracy: A Dynamic Analysis," *American Economic Review,* 88(1): 139-56.
Besley, Timothy and Masayuki Kudamatsu (2009) "Making Autocracy Work," in *Institutions and Economic Performance*, edited by Elhanan Helpman, Harvard University Press, Cambridge.
Brooker, Paul (2000) *Non-Democratic Regimes: Theory, Government, and Politics*, New York: St. Martin's Press.
Bueno de Mesquita, Bruce, Alastair Smith, Randolph D. Siverson and James D. Morrow (2003) *The Logic of Political Survival,* MIT Press, Cambridge.
Caselli, Francesco and Massimo Morelli (2004) "Bad politicians," *Journal of Public Economics,* 88(3-4): 759-782.
Cox, Gary and Jonathan Katz (1996) "Why Did the Incumbency Advantage in U.S. House Elections Grow?" *American Journal of Political Science* 40:478-97.
Chwe, Michael S. Y. (1994) "Farsighted Coalitional Stability," *Journal of Economic Theory,* 63: 299-325.
Diermeier, Daniel, Michael Keane, and Antonio Merlo (2005) "A Political Economy Model of Congressional Careers," *American Economic Review,* 95(1): 347-373.
Egorov, Georgy and Konstantin Sonin (2004) "Dictators and their Viziers: Endogenizing the Loyalty-Competence Trade-off", Harvard, mimeo.
Ferejohn, John (1986) "Incumbent Performance and Electoral Control," *Public Choice,* 50(1): 5-25.
Jones, Benjamin and Benjamin Olken (2005) "Do Leaders Matter? National Leadership and Growth Since World War II," *Quarterly Journal of Economics,* 120(3): 835-864.
Roger Lagunoff (2006) "Markov Equilibrium in Models of Dynamic Endogenous Political Institutions," Georgetown, mimeo.
McKelvey, Richard and Raymond Reizman (1992). "Seniority in Legislatures," *American Political Science Review,* 86(4): 951-965.
Menashri, David (2001) *Post-Revolutionary Politics in Iran: Religion, Society in Power*, Frank Cass Publishers, London.
Meredith, Martin (2007) *Mugabe: Power, Plunder and the Struggle for Zimbabwe*, Perseus Books.
Messner, Matthias, and Mattias Polborn (2004) "Voting on Majority Rules," *Review of Economic Studies,* 71(1): 115-132.

Minier, Jenny (1998) "Democracy and Growth: Alternative Approaches," *Journal of Economic Growth*, 3(3): 241-266.

Moldovanu, Benny, and Eyal Winter (1995) "Order Independent Equilibria," *Games and Economic Behavior*, 9(1), 21-34.

Niskanen William (1971) *Bureaucracy and Representative Government,* New York City, Auldine, Atherton.

Osborne, Martin and Al Slivinski (1996) "A Model of Political Competition with Citizen-Candidates," *Quarterly Journal of Economics*, 111(1): 65-96.

Padro-i-Miquel, Gerard (2007) "The Control of Politicians in Divided Societies: The Politics of Fear." *Review of Economic Studies* 74(4): 1259-1274.

Person, Torsten, Gerard Roland, and Guido Tabellini (1997) "Separation of Power and Political Accountability," *Quarterly Journal of Economics*, 112(4): 1163-1202.

Przeworski, Adam and Fernando Limongi. (1997) "Modernization: Theories and Facts." *World Politics* 49(1): 155-183.

Ray, Debraj (2008) *A Game-Theoretic Perspective on Coalition Formation*, Oxford University Press.

Roberts, Kevin (1999) "Dynamic Voting in Clubs," London School of Economics, mimeo.

Shleifer, Andrei and Robert Vishny (1993) "Corruption," *Quarterly Journal of Economics*, 108(3): 599-617.